EXAMPLE CLASS

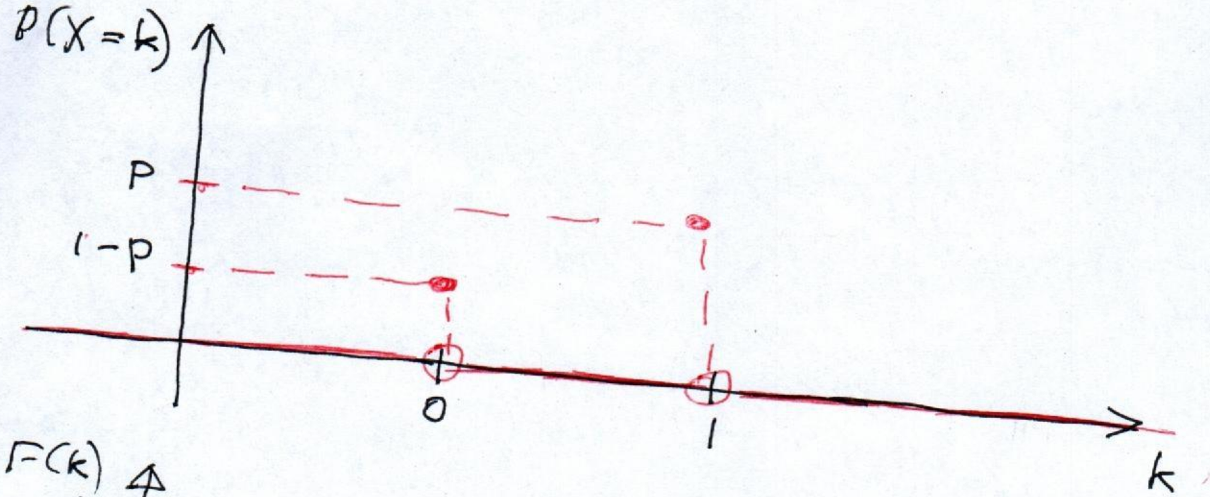
16 OCTOBER

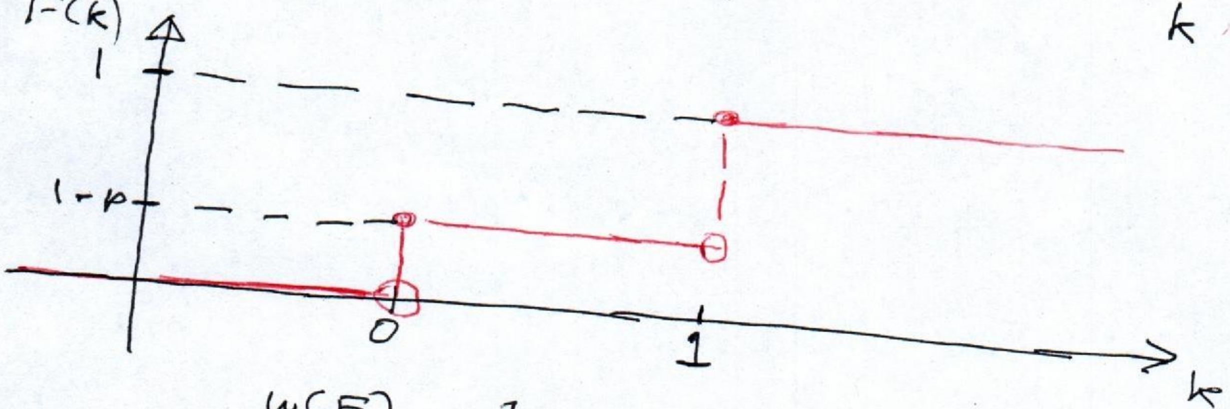
16:00-17:00PM

MATH3/4/68181

Q1 $X \sim \text{Bernoulli}(p)$

$$X = \begin{cases} 1 & \text{with prob } p \\ 0 & \text{" " } 1-p \end{cases}$$





$w(F) = 1$

$$\lim_{k \to 1} \frac{P(X=k)}{1 - F(k-1)} = \frac{P(X=1)}{1 - F(1-1)}$$

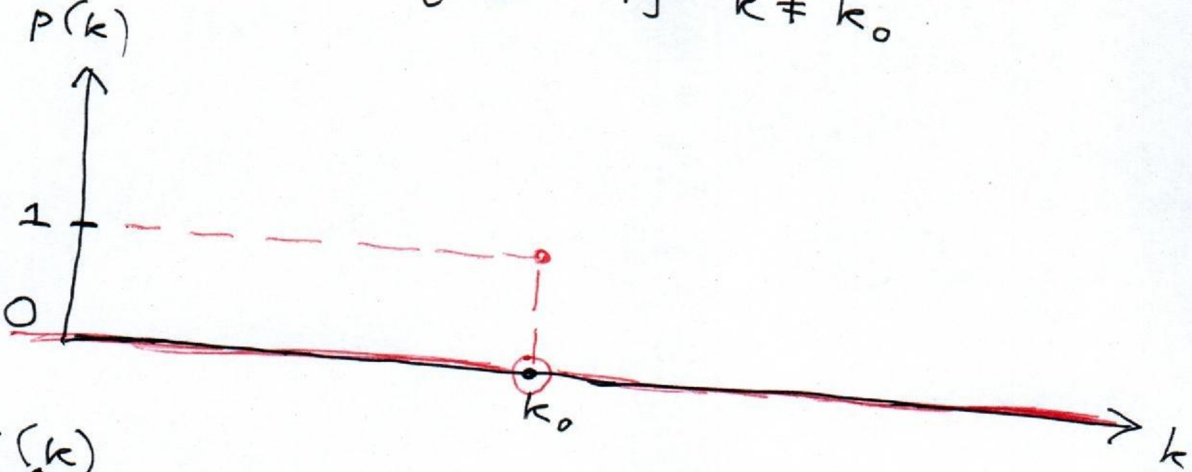
$$= \frac{p}{1 - F(0)} = \frac{p}{1-(1-p)}$$

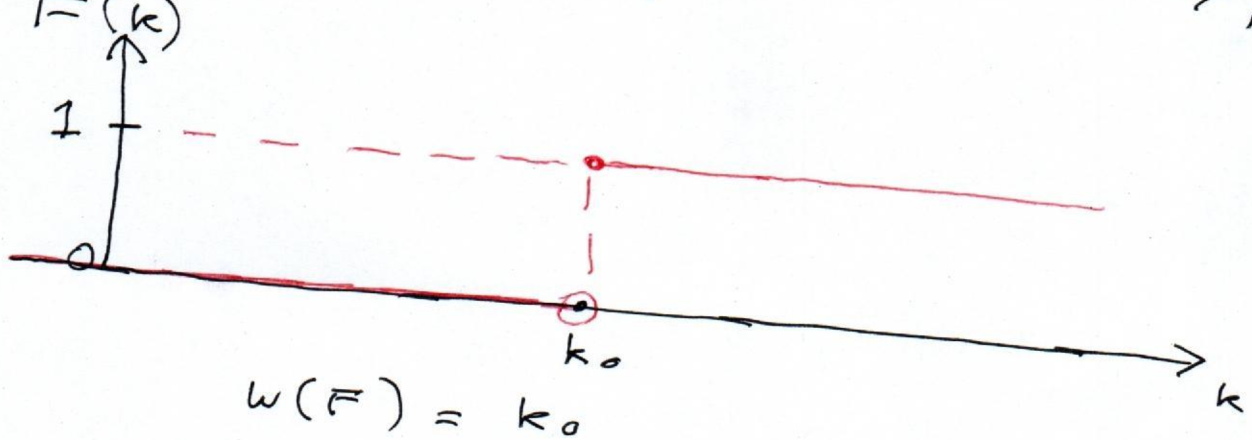
$$= \frac{p}{p} = 1 \neq 0$$

$\Rightarrow$ There exist no sequences $a_n > 0$ and $b_n \in \mathbb{R}$ such that ETT holds.

Q2

$$p(k) = \begin{cases} 1 & \text{if } k = k_0 \\ 0 & \text{if } k \neq k_0 \end{cases}$$





$$w(F) = k_0$$

$$\lim_{k \to k_0} \frac{P(X=k)}{1-F(k-1)}$$

$$= \frac{P(X=k_0)}{1-F(k_0-1)}$$

$$= \frac{1}{1-0} = 1 \neq 0$$

There exists no sequences $a_n > 0$ and $b_n \in \mathbb{R}$ such that ETT holds.

Q4

$$p(k) = \frac{k^{-s}}{\zeta(s)}, \quad k = 1, 2, \ldots$$

$$w(F) = \infty$$

$$\lim_{k \to \infty} \frac{P(X=k)}{1 - F(k-1)}$$

$$= \lim_{k \to \infty} \frac{P(X=k)}{1 - P(X \leq k-1)}$$

$$= \lim_{k \to \infty} \frac{P(X=k)}{P(X \geq k)}$$

$$= \lim_{k \to \infty} \frac{P(X=k)}{\sum_{i=k}^{\infty} P(X=i)}$$

$$= \lim_{k \to \infty} \frac{\frac{k^{-s}}{\cancel{\zeta(s)}}}{\sum_{i=k}^{\infty} \frac{i^{-s}}{\cancel{\zeta(s)}}} = \lim_{k \to \infty} \frac{k^{-s}}{\sum_{i=k}^{\infty} i^{-s}}$$

$$= \lim_{k \to \infty} \frac{k^{-s}}{\int_k^{\infty} x^{-s}\, dx} = \lim_{k \to \infty} \frac{k^{-s}}{\left[\frac{x^{1-s}}{1-s}\right]_k^{\infty}}$$

$$= \lim_{k \to \infty} \frac{k^{-s}}{0 - \frac{k^{1-s}}{1-s}} \qquad \text{provided } s > 1$$

$$= \lim_{k \to \infty} \frac{s-1}{k} = 0 \Rightarrow \text{ETT will hold.}$$

Q 5 $\quad p(k) = -\log\left[1 - \frac{1}{(k+1)^2}\right], \quad k = 1, 2, \ldots$

$w(F) = \infty$

$$\lim_{k\to\infty} \frac{P(X=k)}{1-F(k-1)}$$

$$= \lim_{k\to\infty} \frac{-\log_2\left[1 - \frac{1}{(k+1)^2}\right]}{\cancel{X} - \left[\cancel{X} - \log_2\left[\frac{k+1}{k}\right]\right]}$$

$$= \lim_{k\to\infty} \frac{-\log_2\left[\frac{k^2+2k}{(k+1)^2}\right]}{\log_2\left[\frac{k+1}{k}\right]}$$

$$= \lim_{k\to\infty} \frac{-\log_2(k^2+2k) + 2\log_2(k+1)}{\log_2(k+1) - \log_2 k}$$

$$\stackrel{LH}{=} \lim_{k\to\infty} \frac{-\frac{2k+2}{\cancel{(\log 2)}\cdot(k^2+2k)} + \frac{2}{\cancel{(\log 2)}(k+1)}}{\frac{1}{\cancel{(\log 2)}(k+1)} - \frac{1}{\cancel{(\log 2)}\,k}}$$

$$\boxed{\frac{\partial}{\partial x}\log_2 x = \frac{1}{(\log 2)\,x}}$$

$$= \lim_{k\to\infty} \frac{-\frac{2(k+1)}{k(k+2)} + \frac{2}{k+1}}{\frac{1}{k+1} - \frac{1}{k}}$$

$$= \lim_{k\to\infty} \frac{\frac{-2(k+1)^2 + 2k(k+2)}{k(k+2)(k+1)}}{\frac{-1}{(k+1)k}}$$

$$= \lim_{k\to\infty} \frac{\frac{-2}{k(k+2)(k+1)}}{\frac{-1}{(k+1)k}} = \lim_{k\to\infty} \frac{2}{k+2} = 0$$

$\Rightarrow$ ETT will hold

Q7

$$w(F) = \infty$$

$$\lim_{k \to \infty} \frac{P(X = k)}{1 - F(k-1)}$$

$$= \lim_{k \to \infty} \frac{F(k) - F(k-1)}{1 - F(k-1)}$$

$$\boxed{\begin{array}{c} P(X = k) \\ = F(k) - F(k-1) \end{array}}$$