EXAMPLE CLASS

15 OCTOBER

12:00-13:00PM

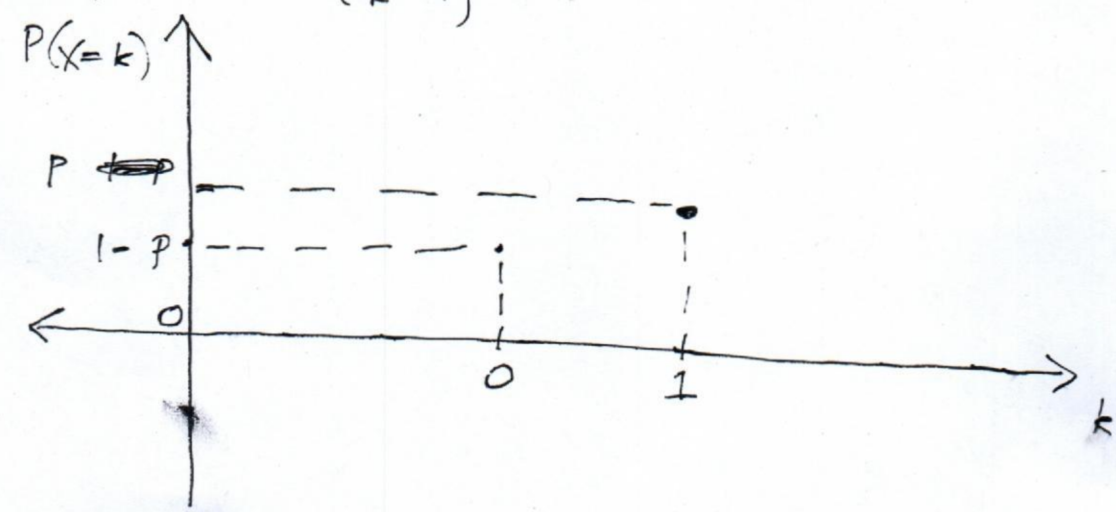
MATH3/4/68181

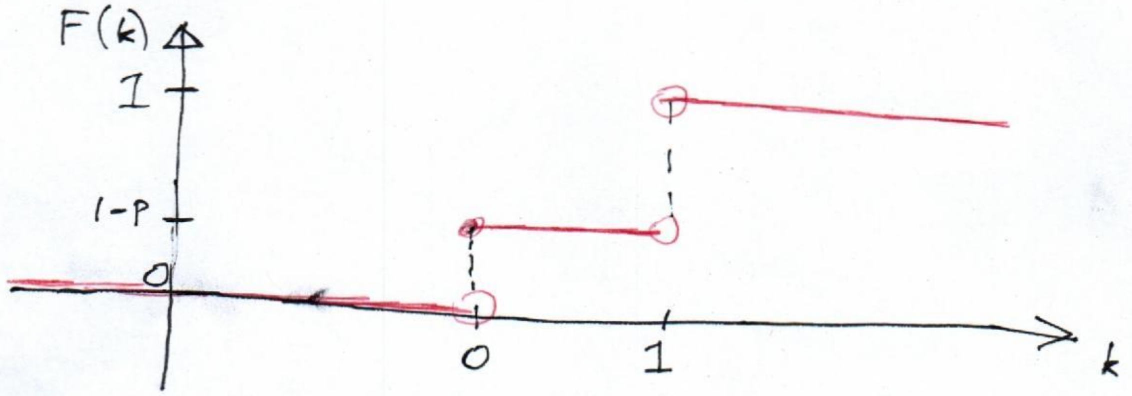
Q1

Bernoulli(p)

$$X = \begin{cases} 1 & \text{with prob } p \\ 0 & \text{" " } 1-p \end{cases}$$

$$\lim_{k \to w(F)} \frac{P(X=k)}{1-F(k-1)}$$



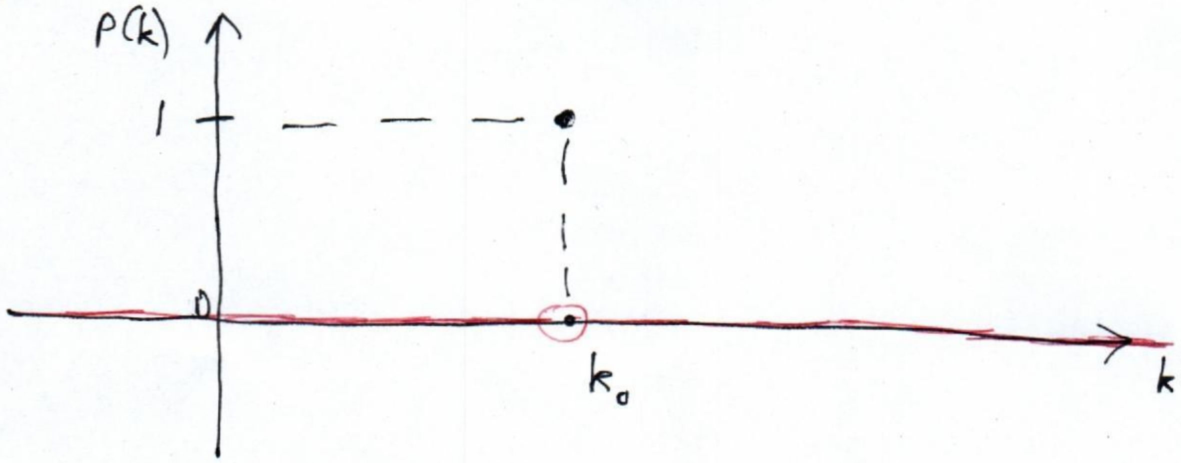


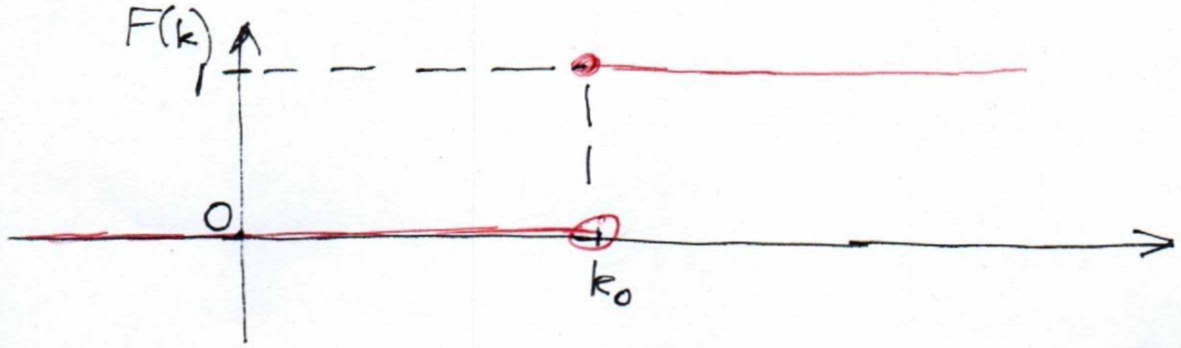
$w(F) = 1$

$$= \frac{P(X=1)}{1-F(1-1)} = \frac{p}{1-F(0)} = \frac{p}{1-(1-p)}$$
$$= 1 \neq 0$$

$\Rightarrow$ There exist no sequences $a_n > 0$ and $b_n \in \mathbb{R}$ such that ETT holds.

Q2





$$\omega(F) = k_0$$

$$\lim_{k \to k_0} \frac{P(X=k)}{1-F(k-1)} = \frac{P(X=k_0)}{1-F(k_0-1)}$$

$$= \frac{1}{1-0} = 1 \neq 0$$

$\Rightarrow$ There exist no sequences $a_n > 0$ and $b_n \in \mathbb{R}$ such that ETT holds.

Q4

$$p(k) = \frac{k^{-s}}{\zeta(s)}, \quad k = 1, 2, \ldots$$

$$\omega(F) = \infty$$

$$\lim_{k\to\infty} \frac{P(X=k)}{1 - F(k-1)}$$

$$= \lim_{k\to\infty} \frac{P(X=k)}{1 - P(X \leq k-1)}$$

$$= \lim_{k\to\infty} \frac{P(X=k)}{P(X \geq k)}$$

$$= \lim_{k\to\infty} \frac{P(X=k)}{\sum_{i=k}^{\infty} P(X=i)}$$

$$= \lim_{k\to\infty} \frac{k^{-s}/\cancel{\zeta(s)}}{\sum_{i=(k)}^{\infty} i^{-s}/\cancel{\zeta(s)}}$$

$$= \lim_{k\to\infty} \frac{k^{-s}}{\sum_{i=(k)}^{\infty} i^{-s}} = \lim_{k\to\infty} \frac{k^{-s}}{\int_{k}^{\infty} x^{-s}\, dx}$$

$$= \lim_{k\to\infty} \frac{k^{-s}}{\left[\frac{x^{1-s}}{1-s}\right]_k^{\infty}} = \lim_{k\to\infty} \frac{k^{-s}}{0 - \frac{k^{1-s}}{1-s}} \quad \text{if } s > 1$$

$$= \lim_{k\to\infty} \frac{s-1}{k} = 0 \Rightarrow \text{ETT holds.}$$

Q5

$$P(k) = -\log_2\left[1 - \frac{1}{(k+1)^2}\right], \quad k = 1, 2, \ldots$$

$$w(F) = \infty.$$

$$\lim_{k\to\infty} \frac{P(X=k)}{1-F(k-1)}$$

$$= \lim_{k\to\infty} \frac{-\log_2\left[1 - \frac{1}{(k+1)^2}\right]}{\left[\cancel{1} - \left[\cancel{1} - \log_2\left[\frac{k+1}{k}\right]\right]\right]}$$

$$= \lim_{k\to\infty} \frac{-\log_2\left[\frac{k^2+2k}{(k+1)^2}\right]}{\log_2\left[\frac{k+1}{k}\right]}$$

$$= \lim_{k\to\infty} \frac{-\log_2[k^2+2k] + 2\log_2[k+1]}{\log_2[k+1] - \log_2 k}$$

$$\stackrel{LH}{=} \lim_{k\to\infty} \frac{-\frac{2k+2}{\cancel{(\log 2)}(k^2+2k)} + \frac{2}{\cancel{(\log 2)}(k+1)}}{\frac{1}{\cancel{(\log 2)}(k+1)} - \frac{1}{\cancel{(\log 2)}\cdot k}}$$

$$\boxed{\frac{\partial \log_2 x}{\partial x} = \frac{1}{(\log 2)\cdot x}}$$

$$= \lim_{k\to\infty} \frac{\frac{-(2k+2)(k+1) + 2(k^2+2k)}{(k^2+2k)(k+1)}}{-\frac{1}{(k+1)k}}$$

$$= \lim_{k\to\infty} \frac{\frac{-1}{(k^2+2k)\cancel{(k+1)}}}{-\frac{1}{\cancel{(k+1)}k}} = \lim -\frac{k}{k^2+2k} = 0$$

$\Rightarrow$ ETT holds.