LECTURE

12 OCTOBER

10:00-11:00AM

MATH4/68181

Financial Ratios

Examples

1) $$\text{Current ratio} = \frac{\text{Current assets (X)}}{\text{" liabilities (Y)}}$$

2) $$\text{Sales margin} = \frac{\text{Sales (X)} - \text{Cost (Y)}}{\text{Sales (X)}}$$

3) $$\text{Interest cover} = \frac{\text{Earnings (X)} + \text{Interest paid (Y)}}{\text{Earnings (X)}}$$

4) $$\text{Liabilities ratio} = \frac{\text{Liabilities (X)}}{\text{Equity (Y)} + \text{Liabities (X)}}$$

The most popular (also the oldest) model for income data is the Pareto distribution.

↑ Italian Economist

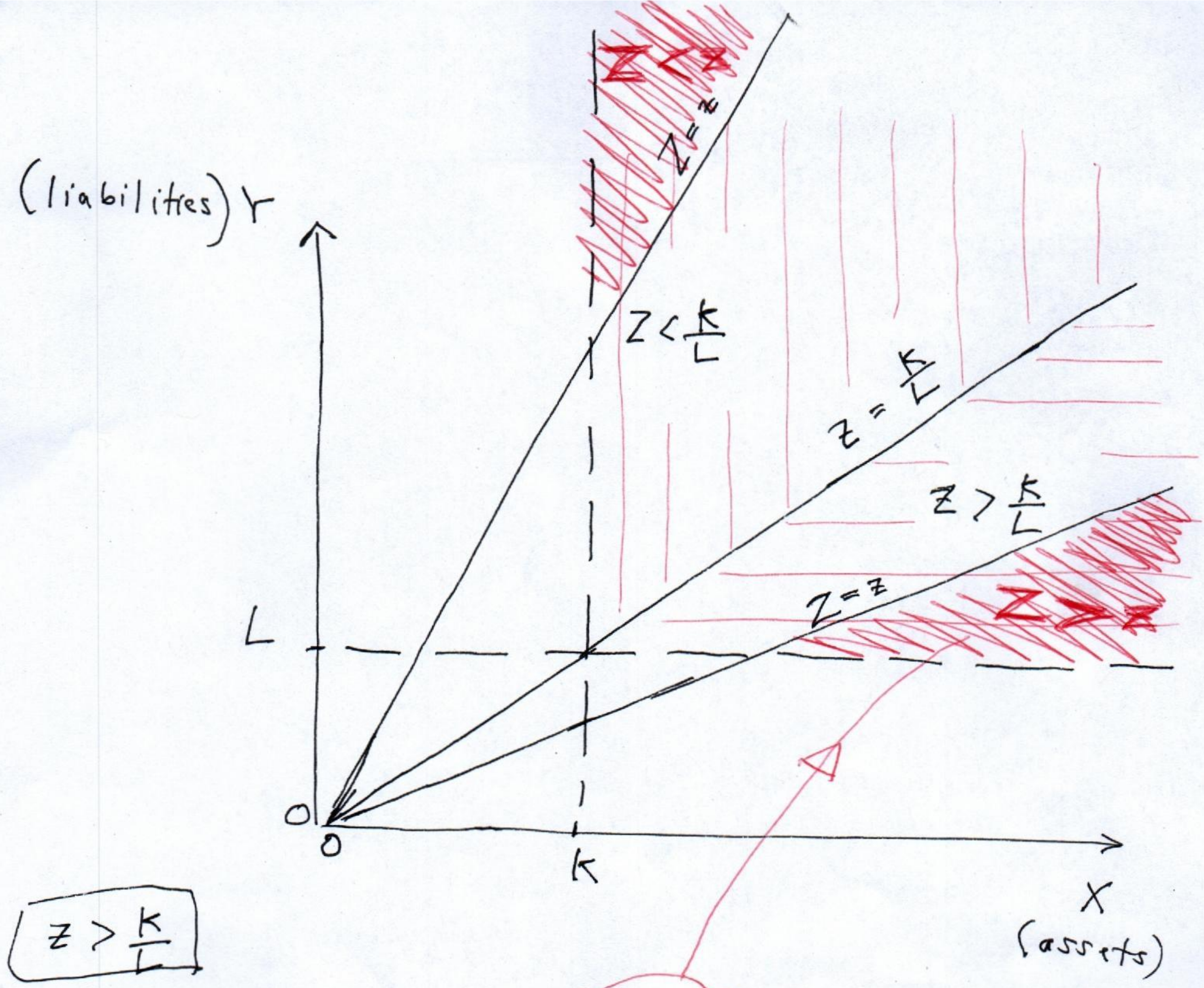
Suppose X and Y are independent Pareto RVs with CDFs

$$F_X(x) = 1 - \left(\frac{k}{x}\right)^a, \quad x \geq k, \; a > 0$$

$$F_Y(y) = 1 - \left(\frac{L}{y}\right)^b, \quad y \geq L, \; b > 0$$

What is the distribution of $Z = \frac{X}{Y}$

↗ Current ratio



$$\boxed{z > \frac{K}{L}}$$

$$F_Z(z) = P(Z \le z) = 1 - P(Z > z)$$

$$= 1 - \int_L^{\infty} \int_{yz}^{\infty} f_X(x) f_Y(y)\, dx\, dy$$

$$= 1 - \int_L^{\infty} \int_{yz}^{\infty} \frac{a K^a}{x^{a+1}} \frac{b L^b}{y^{b+1}}\, dx\, dy$$

$$= 1 - a b K^a L^b \int_L^{\infty} \frac{1}{y^{b+1}} \left[\frac{x^{-a}}{-a}\right]_{yz}^{\infty} dy$$

$$= 1 - a b K^a L^b \int_L^{\infty} \frac{1}{y^{b+1}} \left[0 - \frac{(yz)^{-a}}{-a}\right] dy$$

$$= 1 - b K^a L^b z^{-a} \int_L^{\infty} \frac{1}{y^{a+b+1}}\, dy$$

$$= 1 - b K^a L^b z^{-a} \left[y^{-a-b} / (-a-b)\right]_L^{\infty}$$

$$= 1 - b\, K^{a} L^{b} z^{-a} \left[0 - \frac{L^{-a-b}}{-a-b} \right]$$

$$\boxed{F_Z(z) = 1 - \frac{b}{a+b} K^{a} L^{-a} z^{-a} \quad \text{if} \quad z > \frac{K}{L}}$$

$$\boxed{z < \frac{K}{L}}$$

$$F_Z(z) = P(Z < z)$$

$$= \int_K^\infty \int_{x/z}^\infty \frac{a K^a}{x^{a+1}} \frac{b L^b}{y^{b+1}} \, dy \, dx$$

$$= a b K^a L^b \int_K^\infty \frac{1}{x^{a+1}} \left[\frac{y^{-b}}{-b} \right]_{\frac{x}{z}}^\infty dx$$

$$= a b K^a L^b \int_K^\infty \frac{1}{x^{a+1}} \left[0 - \frac{(x/z)^{-b}}{-b} \right] dx$$

$$= a K^a L^b z^b \int_K^\infty \frac{1}{x^{a+b+1}} \, dx$$

$$= a K^a L^b z^b \left[\frac{x^{-a-b}}{-a-b} \right]_K^\infty$$

$$= a K^a L^b z^b \left[0 - \frac{K^{-a-b}}{-a-b} \right]$$

$$\boxed{= \frac{a}{a+b} K^{-b} L^b z^b \quad \text{if} \quad z < \frac{K}{L}}$$

The CDF of Z is

$$F_Z(z) = \begin{cases} 1 - \frac{b}{a+b}\left(\frac{k}{L}\right)^{a} z^{-a}, & z > \frac{k}{L} \\ \frac{a}{a+b}\left(\frac{L}{k}\right)^{b} z^{b}, & z \leq \frac{k}{L} \end{cases}$$

The PDF of Z is

$$f_Z(z) = \begin{cases} \frac{ab}{a+b}\left(\frac{k}{L}\right)^{a} z^{-a-1} & z > \frac{k}{L} \\ \frac{ab}{a+b}\left(\frac{L}{k}\right)^{b} z^{b-1} & z \leq \frac{k}{L} \end{cases}$$

Median (Current ratio) = ? $F_Z(z) = \frac{1}{2} \Rightarrow z = ?$

Current ratio so extreme expected once in 10 years = ?

$F_Z(z) = 1 - \frac{1}{10} \Rightarrow z = ?$