EXAMPLE CLASS

9 OCTOBER

16:00-17:00PM

MATH3/4/68181

Q 9

$$F(x) = x, \quad 0 < x < 1$$

$$w(F) = 1$$

I: $\lim_{t \to 1} \frac{1 - F(t + x\gamma(t))}{1 - F(t)}$

$$= \lim_{t \to 1} \frac{1 - (t + x\gamma(t))}{1 - t}$$

$$= \lim_{t \to 1} 1 - \frac{x\gamma(t)}{1 - t}$$

$$\neq e^{-x}$$

$\Rightarrow$ cond I is not satisfied

III: $\lim_{t \to 0} \frac{1 - F(1 - tx)}{1 - F(1 - t)}$

$$= \lim_{t \to 0} \frac{1 - (1 - tx)}{1 - (1 - t)}$$

$$= \lim_{t \to 0} \frac{tx}{t}$$

$$= x$$

$\Rightarrow$ Cond III is satisfied with $\alpha = 1$

By ETT, there exists $a_n > 0$ and $b_n \in R$ such that

$$[F(a_n x + b_n)]^n \longrightarrow \begin{cases} e^{x} & x < 0 \\ 1 & x \geq 1 \end{cases}$$

Q10

$$F(x) = 1 - \left(\frac{k}{x}\right)^a, \quad x \geq k$$

$$w(F) = \infty$$

I: $\lim\limits_{t \to \infty} \dfrac{1 - F(t + x\gamma(t))}{1 - F(t)}$

$$= \lim_{t \to \infty} \frac{1 - \left[1 - \left(\frac{k}{t + x\gamma(t)}\right)^a\right]}{1 - \left[1 - \left(\frac{k}{t}\right)^a\right]}$$

$$= \lim_{t \to \infty} \left[\frac{t}{t + x\gamma(t)}\right]^a$$

$$= \lim_{t \to \infty} \left[\frac{1}{1 + x\frac{\gamma(t)}{t}}\right]^a$$

$$\neq e^{-x}$$

$\Rightarrow$ cond (I) is not satisfied

II: $\lim\limits_{t \to \infty} \dfrac{1 - F(tx)}{1 - F(t)}$

$$= \lim_{t \to \infty} \frac{1 - \left[1 - \left(\frac{k}{tx}\right)^a\right]}{1 - \left[1 - \left(\frac{k}{t}\right)^a\right]}$$

$$= \lim_{t \to \infty} \left(\frac{t}{tx}\right)^a$$

$$= x^{-a}$$

$\Rightarrow$ cond (II) is satisfied with $\alpha = +a$

By ETT, there exists $a_n > 0$ and $b_n \in \mathbb{R}$ s.t.

$$\left[F(a_n x + b_n)\right]^n \longrightarrow \begin{cases} e^{-x^{-a}} & x > 0 \\ 0 & x \leq 0 \end{cases}$$

Q11 $f(x) = k\, g(x) \left[G(x) \right]^{a-1} \left[1 - G(x) \right]^{b-1} e^{-c\,G(x)}$

Show that F belongs to the same domain of attraction as G

(i) G belongs to the Gumbel DoA
$\Rightarrow$ F " " " " "

(ii) G belongs to the Frechet DoA
$\Rightarrow$ F " " " " "

(iii) G belongs to the Weibull DoA
F " " " " "

(i) Assume G belongs to Gumbel DoA.

$\exists\ \gamma(t) > 0$ such that

$$\lim_{t \to w(G)} \frac{1 - G(t + x\gamma(t))}{1 - G(t)} = e^{-x} \quad \cdot\cdot \quad (*)$$

We need to show that

$$\lim_{t \to w(F)} \frac{1 - F(t + x\gamma(t))}{1 - F(t)} = e^{-x} \quad \cdot\cdot \quad (**)$$

LHS of $(**)$

$$\overset{LH}{=} \lim_{t \to w(F)} \frac{-f(t + x\gamma(t)) \cdot (1 + x\gamma'(t))}{-f(t)}$$

$$= \lim_{t \to w(F)} \frac{f(t + x\gamma(t))}{f(t)} (1 + x\gamma'(t))$$

$$= \lim_{t \to \underset{w(G)}{\overset{\|}{w(F)}}} \frac{\cancel{K}\, g(t + x\gamma(t)) [G(t + x\gamma(t))]^{a-1} [1 - G(t + x\gamma(t))]^{b-1}}{\cancel{K}\, g(t) [G(t)]^{a-1} [1 - G(t)]^{b-1}}$$

$$\times \frac{e^{-c\, G(t + x\gamma(t))}}{e^{-c\, G(t)}} (1 + x\gamma'(t))$$

$$= \lim_{t \to w(G)} \frac{g(t + x\gamma(t))}{g(t)} \left[\frac{1 - G(t + x\gamma(t))}{1 - G(t)}\right]^{b-1} (1 + x\gamma'(t))$$

$$\overset{(*)}{=} \lim_{t \to w(G)} \frac{g(t + x\gamma(t))}{g(t)} (1 + x\gamma'(t)) \left[e^{-x}\right]^{b-1}$$

$$\underset{\text{backwards}}{\overset{LH}{=}} \lim_{t \to w(G)} \frac{1 - G(t + x\gamma(t))}{1 - G(t)}\, e^{-(b-1)x}$$

$$\stackrel{(*)}{=} \lim_{t \to w(G)} e^{-x} e^{-(b-1)x}$$

$$= e^{-bx}, \text{ the SAME TYPE as } e^{-x}$$

Hence, F also belongs to Gumbe DoA.

(ii) Homework

(iii) "