EXAMPLE CLASS

8 OCTOBER

12:00-13:00PM

MATH3/4/68181

Q9 $F(x) = x, \quad 0 < x < 1$

$\omega(F) = 1$

I : $$\lim_{t \to 1} \frac{1 - F(t + x\gamma(t))}{1 - F(t)}$$

$$= \lim_{t \to 1} \frac{1 - (t + x\gamma(t))}{1 - t}$$

$$= \lim_{t \to 1} \left[1 + \frac{x\gamma(t)}{1 - t}\right]$$

$$\neq e^{-x}$$

$\Rightarrow$ cond (I) is not satisfied

III : $$\lim_{t \to 0} \frac{1 - F(1 - tx)}{1 - F(1 - t)}$$

$$= \lim_{t \to 0} \frac{1 - (1 - tx)}{1 - (1 - t)}$$

$$= \lim_{t \to 0} \frac{tx}{t}$$

$$= x$$

$\Rightarrow$ cond (III) is satisfied with $\alpha = 1$

By the ETT, there exists $a_n > 0$ and $b_n \in \mathbb{R}$ such that $[F(a_n x + b_n)]^n \longrightarrow \begin{cases} e^{x} & x < 0 \\ 1 & x > 0 \end{cases}$

Q10

$$F(x) = 1 - \left(\frac{k}{x}\right)^a, \quad x > k$$

$$\omega(F) = \infty.$$

$$\text{I}: \lim_{t \to \infty} \frac{1 - F(t + x\gamma(t))}{1 - F(t)}$$

$$= \lim_{t \to \infty} \frac{\cancel{1} - \left[\cancel{1} - \left(\frac{k}{t + x\gamma(t)}\right)^a\right]}{\cancel{1} - \left[\cancel{1} - \left(\frac{\cancel{k}}{\cancel{t}}\right)^a\right]}$$

$$= \lim_{t \to \infty} \left(\frac{t}{t + x\gamma(t)}\right)^a$$

$$= \lim_{t \to \infty} \left(\frac{1}{1 + \frac{x\gamma(t)}{t}}\right)^a$$

$$\neq e^{-x}$$

$\Rightarrow$ cond (I) is not satisfied.

II:

$$\lim_{t \to \infty} \frac{1 - F(tx)}{1 - F(t)}$$

$$= \lim_{t \to \infty} \frac{\cancel{1} - \left[\cancel{1} - \left(\frac{k}{tx}\right)^a\right]}{\cancel{1} - \left[\cancel{1} - \left(\frac{k}{t}\right)^a\right]}$$

$$= \lim_{t \to \infty} \frac{t^a}{(tx)^a}$$

$$= x^{-a}$$

$\Rightarrow$ Cond (II) is satisfied with $\alpha = a$.

By the ETT, there exists $a_n > 0$ and $b_n \in \mathbb{R}$ such that

$$[F(a_n x + b_n)]^n \longrightarrow \begin{cases} e^{-x^{-a}} & x > 0 \\ 0 & x \leq 0 \end{cases}$$

as $n \to \infty$.

Q11 Two cdfs F & G.

$$f(x) = K\, g(x) \left[G(x)\right]^{a-1} \left[1-G(x)\right]^{b-1} e^{-c\,G(x)}$$

Show that F belongs to the same DoA as G

(i) G belongs to Gumbel domain
$\Rightarrow$ F " " " "

(ii) G belongs to Frēchet domain
$\Rightarrow$ F " " " "

(iii) G belongs to Weibull domain
$\Rightarrow$ F " " " "

Proof of (i)

Assume G belongs to Gumbel domain.

$\Rightarrow \exists\ \gamma(t) > 0$ such that

$$\lim_{t \to w(G)} \frac{1 - G(t + x\gamma(t))}{1 - G(t)} = e^{-x} \quad \cdots (*).$$

We need to show that

$$\lim_{t \to w(F)} \frac{1 - \overbrace{F(t + x\gamma(t))}^{\to 1}}{1 - \underbrace{F(t)}_{\to 1}} = e^{-x} \quad (**)$$

LHS of $(**)$

$$\overset{LH}{=} \lim_{t \to w(F)} \frac{-f(t + x\gamma(t))\,(1 + x\gamma'(t))}{-f(t)}$$

$$= \lim_{t \to w(F)} \frac{f(t + x\gamma(t))\,(1 + x\gamma'(t))}{f(t)}$$

$$= \lim_{t \to \underbrace{w(F)}_{= w(G)}} \frac{\cancel{k}\, g(t + x\gamma(t)) \left[\overbrace{G(t + x\gamma(t))}^{\to 1}\right]^{a-1} \left[1 - G(t + x\gamma(t))\right]^{b-1} \cancel{e^{-c\overbrace{G(t+x\gamma(t))}^{\to 1}}}}{\cancel{k}\, g(t) \left[\underbrace{G(t)}_{\to 1}\right]^{a-1} \left[1 - G(t)\right]^{b-1} \cancel{e^{-c\underbrace{G(t)}_{\to 1}}}}$$

$$\times\ (1 + x\gamma'(t))$$

$$= \lim_{t \to w(G)} \frac{g(t+x\gamma(t)) \, [1 - G(t+x\gamma(t))]^{b-1}}{g(t) \, [1 - G(t)]^{b-1}} (1 + x\gamma'(t))$$

$$= \lim_{t \to w(G)} \frac{g(t+x\gamma(t))}{g(t)} (1 + x\gamma'(t)) \left[\frac{1 - G(t+x\gamma(t))}{1 - G(t)}\right]^{b-1}$$

$$\overset{(*)}{=} \lim_{t \to w(G)} \frac{g(t+x\gamma(t))}{g(t)} (1 + x\gamma'(t)) \left[e^{-x}\right]^{b-1}$$

$$\underset{\text{in reverse}}{\overset{LH}{=}} \lim_{t \to w(G)} \frac{1 - G(t+x\gamma(t))}{1 - G(t)} \, e^{-(b-1)x}$$

$$\overset{(*)}{=} e^{-x} \, e^{-(b-1)x} = e^{-bx},$$

the SAME TYPE as e^{-x}.

Hence, F also belongs to the Gumbel domain.