

EXAMPLE CLASS

1 OCTOBER

12:00-13:00PM

MATH3/4/68181

Q1

$$\lambda(x) = e^{-e^{-x}}$$

$$\begin{aligned}\lambda'(x) &= e^{-e^{-x}} (-1) e^{-x} (-1) \\ &= e^{-e^{-x}} e^{-x}\end{aligned}$$

~~Q2~~

$$\Phi_\alpha(x) = \begin{cases} 0 & x < 0 \\ e^{-x^{-\alpha}} & x \geq 0 \end{cases}$$

$$\Phi_\alpha'(x) = \begin{cases} 0 & x < 0 \\ e^{-x^{-\alpha}} (-1) (-\alpha) x^{-\alpha-1} & x \geq 0 \end{cases}$$

$$= \begin{cases} 0 & x < 0 \\ \alpha x^{-\alpha-1} e^{-x^{-\alpha}} & x \geq 0 \end{cases}$$

~~Q3~~

$$\Psi_\alpha(x) = \begin{cases} e^{-(-x)^\alpha} & x < 0 \\ 1 & x \geq 0 \end{cases}$$

$$\Psi_\alpha'(x) = \begin{cases} e^{-(-x)^\alpha} (-1) (-x)^{\alpha-1} \alpha \cdot (-1) & x < 0 \\ 0 & x \geq 0 \end{cases}$$

$$= \begin{cases} \alpha (-x)^{\alpha-1} e^{-(-x)^\alpha} & x < 0 \\ 0 & x \geq 0 \end{cases}$$

Q2

$$\Lambda(x) = e^{-e^{-x}}$$

$$\Lambda'(x) = e^{-x} e^{-e^{-x}}$$

$$\text{Mean} = \int_{-\infty}^{\infty} x \cdot e^{-x} e^{-e^{-x}} dx$$

$$\boxed{\begin{aligned} \text{Set } y &= e^{-x} \Rightarrow x = -\log y \\ \frac{dx}{dy} &= -\frac{1}{y} \end{aligned}}$$

$$= \int_{\infty}^0 (-\log y) y e^{-y} \left(-\frac{1}{y}\right) dy$$

$$= - \int_0^{\infty} \log y e^{-y} dy$$

$$\boxed{\begin{aligned} \frac{d}{da} y^a &= y^a \log y \\ \left. \frac{dy^a}{da} \right|_{a=0} &= y^0 \log y = \log y \end{aligned}} \quad (*)$$

$$\stackrel{(*)}{=} - \int_0^{\infty} \left. \frac{dy^a}{da} \right|_{a=0} e^{-y} dy$$

$$= - \frac{d}{da} \left[\int_0^{\infty} y^a e^{-y} dy \right] \Big|_{a=0}$$

$$= - \frac{d}{da} \Gamma(a+1) \Big|_{a=0}$$

$$= - \Gamma'(a+1) \Big|_{a=0}$$

$$= - \Gamma'(1)$$

$$\boxed{\begin{aligned} \Gamma(a) &= \int_0^{\infty} t^{a-1} e^{-t} dt \\ \text{GAMMA FUNCTION} \end{aligned}}$$

Q3

$$\Lambda(x) = e^{-e^{-x}}$$

$$\Lambda'(x) = e^{-x} e^{-e^{-x}}$$

$$\text{Variance} = E(X^2) - (E(X))^2$$

$$= E(X^2) - [\Gamma'(1)]^2$$

$$E(X^2) = \int_{-\infty}^{\infty} x^2 e^{-x} e^{-e^{-x}} dx$$

$$\text{Set } y = e^{-x} \Rightarrow x = -\log y \\ \Rightarrow \frac{dx}{dy} = -\frac{1}{y}$$

$$= \int_{\infty}^0 (\log y)^2 y e^{-y} \left(-\frac{1}{y}\right) dy$$

$$= \int_0^{\infty} (\log y)^2 e^{-y} dy$$

$$\frac{d^a y^a}{da^a} = y^a \log y \\ \Rightarrow \frac{d^2 y^a}{da^2} = y^a (\log y)^2 \\ \Rightarrow \frac{d^2 y^a}{da^2} \Big|_{a=0} = y^0 (\log y)^2 = (\log y)^2 \quad (*)$$

$$(*) \int_0^{\infty} \frac{d^2 y^a}{da^2} \Big|_{a=0} e^{-y} dy$$

$$= \frac{d^2}{da^2} \left[\int_0^{\infty} y^a e^{-y} dy \right] \Big|_{a=0}$$

$$= \frac{d^2}{da^2} [\Gamma(a+1)] \Big|_{a=0} = \Gamma''(1)$$

$$\text{Variance} = \Gamma''(1) - [\Gamma'(1)]^2.$$

Q4

$$\Lambda^n(x) = \Lambda(\alpha_n x + \beta_n)$$

$$\Leftrightarrow [e - e^{-x}]^n = e - e^{-\alpha_n x - \beta_n}$$

$$\Leftrightarrow e - n e^{-x} = e - e^{-\alpha_n x - \beta_n}$$

Take log

$$\Leftrightarrow -n e^{-x} = -e^{-\alpha_n x - \beta_n}$$

$$\Leftrightarrow n e^{-x} = e^{-\alpha_n x - \beta_n}$$

Take log

$$\Leftrightarrow \log n - x = -\alpha_n x - \beta_n$$

$$\Leftrightarrow \begin{aligned} -1 &= -\alpha_n \\ \& \log n &= -\beta_n \end{aligned}$$

$$\Leftrightarrow \alpha_n = 1 \text{ and } \beta_n = -\log n.$$

~~https://~~

<https://minerva.it.manchester.ac.uk/~saralees/>

extremes3.html

extremes4.html

extremes6.html

Q7

Done in lectures

Q8

$$F(x) = [1 - e^{-x}]^x$$

$$F(x) = 1 \Rightarrow [1 - e^{-x}]^x = 1$$

$$\Rightarrow 1 - e^{-x} = 1 \Rightarrow e^{-x} = 0 \Rightarrow x = \infty$$

$$\Rightarrow w(F) = \infty$$

$$I: \lim_{t \rightarrow \infty} \frac{1 - F(t + x\gamma(t))}{1 - F(t)}$$

$$= \lim_{t \rightarrow \infty} \frac{1 - [1 - e^{-t - x\gamma(t)}]^x}{1 - [1 - e^{-t}]^x}$$

$$(1 - a)^x \approx 1 - xa \text{ as } a \rightarrow 0$$

$$= \lim_{t \rightarrow \infty} \frac{1 - [1 - x e^{-t - x\gamma(t)}]}{1 - [1 - x e^{-t}]}$$

$$= \lim_{t \rightarrow \infty} \frac{e^{-t - x\gamma(t)}}{e^{-t}}$$

$$= \lim_{t \rightarrow \infty} e^{-x\gamma(t)}$$

$$= e^{-x} \text{ if } \gamma(t) \equiv 1 \quad \forall t$$

By the ETT, there exist $a_n > 0$ and $b_n \in \mathbb{R}$ such that

$$[F(a_n x + b_n)]^n \rightarrow e^{-e^{-x}}$$

as $n \rightarrow \infty$.