REVISION CLASS

11 DECEMBER

9:00-10:00AM

MATH3/4/68181

REVISION

CLASS

TUESDAY 11 DEC

9:00- 10:00 AM

Q2, Exam 2017/18

$$f(x) = c\, g(x) \left[1 - G(x)\right]^{\lambda b - 1} \times \left\{1 - \left[1 - G(x)\right]^{\lambda}\right\}^{a-1}$$

Show that F and G belong to the same domain of attraction.

(i) Assume G belongs to the Gumbel max domain. There exists $\gamma(t) > 0$ such that

$$\lim_{t \to w(G)} \frac{1 - G(t + x\gamma(t))}{1 - G(t)} = e^{-x} \quad \ldots (*).$$

$$\lim_{t \to w(F)} \frac{1 - F(t + x\gamma(t))}{1 - F(t)}$$

$$\stackrel{LH}{=} \lim_{t \to w(F)} \frac{-f(t + x\gamma(t))\,(1 + x\gamma'(t))}{-f(t)}$$

$$= \lim_{t \to w(F)} \frac{f(t + x\gamma(t))\,(1 + x\gamma'(t))}{f(t)}$$

$$= \lim_{t \to w(G)} \frac{\cancel{C}\, g(t + x\gamma(t))}{\cancel{C}\, g(t)} \left[\frac{1 - G(t + x\gamma(t))}{1 - G(t)} \right]^{\lambda b - 1}$$

$$\times \left\{ \frac{1 - [1 - G(t + x\gamma(t))]^{\lambda}}{1 - [1 - G(t)]^{\lambda}} \right\}^{a-1}$$

$$\times (1 + x\gamma'(t))$$

$$\to 1 \quad \to 1 \quad \to 1 \quad \to 1 \quad \to 1$$

$$= \lim_{t \to w(G)} \frac{g(t + x\gamma(t))}{g(t)} \cdot (1 + x\gamma'(t))$$

$$\times \left[\frac{1 - G(t + x\gamma(t))}{1 - G(t)} \right]^{\lambda b - 1}$$

$$\overset{LH}{=} \lim_{t \to w(G)} \frac{1 - G(t + x\gamma(t))}{1 - G(t)} \cdot \left[\frac{1 - G(t + x\gamma(t))}{1 - G(t)} \right]^{\lambda b - 1}$$

in reverse

$$= \lim_{t \to w(G)} \left[\frac{1 - G(t + x\gamma(t))}{1 - G(t)} \right]^{\lambda b}$$

$$\overset{(*)}{=} \left[e^{-x} \right]^{\lambda b} = e^{-\lambda b x}$$

$\Rightarrow$ F also belongs to the Gumbel max domain.

Level 3

2 hrs

5 Q

must answer any 4 Q

Levels 4 & 6

3 hrs

Section A

- 3 Q
- must answer any 2 Q

Section B

- 5 Q
- must answer any 4 Q

Q3 (e), 2017/18 Exam

$$F(x) = \left[1 + e^{-ax}\right]^{-b}, \quad -\infty < x < \infty$$

$$w(F) = +\infty$$

$$I: \lim_{t \to \infty} \frac{1 - \left[1 + e^{-a(t + x\gamma(t))}\right]^{-b}}{1 - \left[1 + e^{-at}\right]^{-b}}$$

$\to 0$ $\quad$ $\to 0$

$$= \lim_{t \to \infty} \frac{\cancel{1} - \left[\cancel{1} - \cancel{b}\, e^{-a(t + x\gamma(t))}\right]}{\cancel{1} - \left[\cancel{1} - \cancel{b}\, e^{-at}\right]}$$

$$\boxed{(1+z)^{-b} \sim 1 - bz}$$

$$= \lim_{t \to \infty} \frac{e^{-a(t + x\gamma(t))}}{e^{-at}}$$

$$= \lim_{t \to \infty} e^{-ax\gamma(t)}$$

$$= e^{-x} \quad \text{if} \quad \gamma(t) \equiv \frac{1}{a}$$

Hence, F belongs to the Gumbel max domain.

Q4(b), Exam 2017/18

$X_i \sim N(\mu_i, \sigma_i^2)$ independently

(i) $T = X_1 + X_2 + \cdots + X_k$

$$\sim N\left(\sum_{i=1}^{k} \mu_i , \sum_{i=1}^{k} \sigma_i^2\right)$$

(ii) $F_T(t) = p$

$$\Rightarrow \Phi\left(\frac{t - \sum_{i=1}^{k} \mu_i}{\sqrt{\sum_{i=1}^{k} \sigma_i^2}}\right) = p$$

$$\Rightarrow \mathrm{VaR}_p(T) = \left(\sum_{i=1}^{k} \mu_i\right) + \sqrt{\sum_{i=1}^{k} \sigma_i^2}\, \Phi^{-1}(p)$$

(iii) $$ES_p(T) = \frac{1}{p} \int_0^p \mathrm{VaR}_u(T)\, du$$

$$= \left(\sum_{i=1}^{k} \mu_i\right) + \sqrt{\sum_{i=1}^{k} \sigma_i^2}\ \frac{1}{p} \int_0^p \Phi^{-1}(u)\, du.$$

Q 2 (b), 2017 / 18 Exam

$$p(k) = \frac{\binom{K}{k}\binom{N-K}{n-k}}{\binom{N}{n}}$$

$$k = \max(0, n+K-N), \ldots, \min(n, K).$$

$$\omega(F) = \min(n, K)$$

$$\lim_{k \to \omega(F)} \frac{P(X = k)}{1 - F(k-1)} = \frac{P(X = \min(n, K))}{1 - F(\min(n, K) - 1)}$$

$$= \frac{P(X = \min(n, K))}{1 - P(X \leq \min(n, K) - 1)}$$

$$= \frac{P(X = \min(n, K))}{P(X \geq \min(n, K))}$$

$$= \frac{P(X = \min(n, K))}{P(X = \min(n, K))} = 1 \neq 0$$

Hence, ETT does not hold.