REVISION CLASS

14 DECEMBER

10:00-11:00AM

MATH4/68181

REVISION

CLASS

TUESDAY 11 DEC

4 - 5 PM

Q2 (c) Exam 2017/18

$$f(x) = C e^{\beta x - \eta e^{bx}}, \quad x > 0$$

$$\omega(F) = +\infty$$

I: $\lim_{t \to \infty} \dfrac{1 - F(t + x\gamma(t))}{1 - F(t)}$

$$\overset{LH}{=} \lim_{t\to\infty} \frac{-f(t+x\gamma(t))\,(1+x\gamma'(t))}{-f(t)}$$

$$= \lim_{t\to\infty} \frac{\cancel{C} e^{\boxed{\beta(t+x\gamma(t))} - \eta e^{b(t+x\gamma(t))}} (1+x\gamma'(t))}{\cancel{C} e^{\boxed{\beta t} - \eta e^{bt}}}$$

$$= \lim_{t\to\infty} e^{\beta x \gamma(t)} e^{\eta e^{bt} - \eta e^{b(t+x\gamma(t))}} (1+x\gamma'(t))$$

$$= \lim_{t\to\infty} e^{\beta x\gamma(t)} e^{\eta e^{bt}\left[1 - e^{bx\gamma(t)}\right]} (1+x\gamma'(t))$$

Assume $\gamma(t) \to 0$

$e^{z} \sim 1 + z$

$$= \lim_{t\to\infty} e^{\beta x\gamma(t)} e^{\eta e^{bt}\left[\cancel{1} - \cancel{1} - bx\gamma(t)\right]} (1+x\gamma'(t))$$

$$= \lim_{t\to\infty} e^{\beta x \gamma(t)} e^{-\eta b x e^{bt}\gamma(t)} (1 + x\gamma'(t))$$

$0 \leftarrow \beta x\gamma(t)$; $\eta b e^{bt}\gamma(t) = 1$; $x\gamma'(t) \to 0$

$\eta b e^{bt} \gamma(t) = 1$

$\Rightarrow \gamma(t) = \dfrac{1}{\eta b e^{bt}} \longrightarrow 0$ as $t \to \infty$

$\gamma'(t) = -\dfrac{1}{\eta e^{bt}} \longrightarrow 0$ as $t \to \infty$

$$= \lim_{t \to \infty} e^{0} e^{-x} (1+0)$$

$$= e^{-x}$$

Hence, cond I is satisfied and F belongs to the Gumbel domain.

Q 3 (d) Exam 2015/16

$$f(x) = \frac{1}{2} e^{-|x|} \quad , -\infty < x < \infty$$

$$w(F) = \infty$$

$$\text{I} : \lim_{t \to \infty} \frac{1 - F(t + x\gamma(t))}{1 - F(t)}$$

$$\stackrel{LH}{=} \lim_{t \to \infty} \frac{-f(t + x\gamma(t))\,(1 + x\gamma'(t))}{-f(t)}$$

$$= \lim_{t \to \infty} \frac{\cancel{\tfrac{1}{2}}\, e^{-|t + x\gamma(t)|}\,(1 + x\gamma'(t))}{\cancel{\tfrac{1}{2}}\, e^{-|t|}}$$

$$= \lim_{t \to \infty} \frac{e^{-t - x\gamma(t)}\,(1 + x\gamma'(t))}{e^{-t}}$$

$$= \lim_{t \to \infty} e^{-x\gamma(t)}\,(1 + x\gamma'(t))$$

$$\boxed{\begin{array}{l} \gamma(t) \equiv 1 \\ \gamma'(t) \equiv 0 \end{array}}$$

$$= e^{-x}$$

Hence, condition I is satisfied and F belongs to the Gumbel max domain.

Q3(e) Exam 2016/17

$$F(x) = 1 - \left[1 - e^{-\frac{1}{x}}\right]^{a}, \quad x > 0$$

$$\omega(F) = \infty.$$

II : $\displaystyle \lim_{t \to \infty} \frac{1 - F(tx)}{1 - F(t)}$

$$= \lim_{t\to\infty} \frac{\cancel{1} - \{\cancel{1} - [1 - e^{-\frac{1}{tx}}]^{a}\}}{\cancel{1} - \{\cancel{1} - [1 - e^{-\frac{1}{t}}]^{a}\}}$$

$$= \lim_{t\to\infty} \left[\frac{1 - e^{-\frac{1}{tx}}}{1 - e^{-\frac{1}{t}}}\right]^{a}$$

$\frac{1}{tx} \to 0$, $\frac{1}{t} \to 0$

$\boxed{e^{-z} \sim 1 - z}$ as $z \to 0$

$$= \lim_{t\to\infty} \left[\frac{\cancel{1} - (\cancel{1} - \frac{1}{tx})}{\cancel{1} - (\cancel{1} - \frac{1}{t})}\right]^{a}$$

$$= x^{-a}$$

Hence, condition II is satisfied and F belongs to the Fréchet max domain.

Q 3 (c) Exam 2016/17

$$f(x) = \frac{1}{(\log b - \log a)\, x}, \quad 0 < a < x < b < \infty$$

$$w(F) = b$$

III : $$\lim_{t \to 0} \frac{1 - F(b - tx)}{1 - F(b - t)}$$

$$\overset{LH}{=} \lim_{t \to 0} \frac{-f(b - tx) \cdot (-x)}{-f(b - t) \cdot (-1)}$$

$$= \lim_{t \to 0} \frac{\cancel{\frac{1}{(\log b - \log a)}} \frac{1}{b - tx} \cdot x}{\cancel{\frac{1}{(\log b - \log a)}} \frac{1}{b - t}}$$

$\to b$

$\to b$

$$= x$$

Hence, condition III is satisfied and F belongs to the Weibull max domain.

Q3(a) Exam 2015/16

$$f(x) = C\, x^{\alpha-1}(1-x)^{\beta-1}, \quad 0 < x < 1$$

$$w(F) = 1$$

III : $\displaystyle \lim_{t\to 0} \frac{1-F(1-tx)}{1-F(1-t)}$

$$\overset{LH}{=} \lim_{t\to 0} \frac{-f(1-tx)\cdot(-x)}{-f(1-t)\cdot(-1)}$$

$$= \lim_{t\to 0} \frac{f(1-tx)\cdot x}{f(1-t)}$$

$$= \lim_{t\to 0} \frac{\cancel{C}\,\underbrace{(1-tx)^{\alpha-1}}_{\to 1}(tx)^{\beta-1}\,x}{\cancel{C}\,\underbrace{(1-t)^{\alpha-1}}_{\to 1}\,t^{\beta-1}}$$

$$= x^{\beta-1}\cdot x = x^{\beta}$$

$\Rightarrow$ cond III is satisfied and F belongs to the Weibull max domain.