# REVISION CLASS

# 10 DECEMBER

# 12:00-13:00PM

# MATH3/4/68181

REVISION
CLASS
MON 10 DEC
12 - 1 PM

# Quiz 4

$$X = \min(Y_1, Y_2, \ldots, Y_N)$$

$$Y_i = \max(Z_{i,1}, Z_{i,2}, \ldots, Z_{i,\alpha})$$

The CDF of $Y_i$ is

$$F_{Y_i}(y) = P[Y_i \leqslant y]$$

$$= P[\max(Z_{i,1}, \ldots, Z_{i,\alpha}) \leqslant y]$$

$$= P[Z_{i,1} \leqslant y, \ldots, Z_{i,\alpha} \leqslant y]$$

$$\overset{\text{indep}}{=} P[Z_{i,1} \leqslant y] \cdots P[Z_{i,\alpha} \leqslant y]$$

$$= (1 - e^{-\beta y}) \cdots (1 - e^{-\beta y})$$

$$= (1 - e^{-\beta y})^{\alpha}.$$

The CDF of $X$ is

$$F_X(x) = P(X \leqslant x)$$

$$= 1 - P(X > x)$$

total prob rule

$$= 1 - \sum_{n=1}^{\infty} P(X > x / N = n)\, P(N = n)$$

$$= 1 - \sum_{n=1}^{\infty} P(\min(Y_1, \ldots, Y_N) > x / N = n)\, P(N = n)$$

$$= 1 - \sum_{n=1}^{\infty} P(Y_1 > x, \ldots, Y_n > x)\, P(N = n)$$

$$\overset{\text{indep}}{=} 1 - \sum_{n=1}^{\infty} P(Y_1 > x) \cdots P(Y_n > x)\, P(N=n)$$

$$= 1 - \sum_{n=1}^{\infty} [P(Y > x)]^n \, P(N=n)$$

$$= 1 - \sum_{n=1}^{\infty} \left[1-(1-e^{-\beta x})^\alpha\right]^n \frac{1}{e^\lambda - 1} \frac{\lambda^n}{n!}$$

$$= 1 - \frac{1}{e^\lambda - 1} \sum_{n=1}^{\infty} \frac{\{[1-(1-e^{-\beta x})^\alpha]\lambda\}^n}{n!}$$

$$\boxed{\sum_{n=1}^{\infty} \frac{\theta^n}{n!} = e^\theta - 1}$$

$$= 1 - \frac{1}{e^\lambda - 1} \left\{ e^{[1-(1-e^{-\beta x})^\alpha]\lambda} - 1 \right\}$$

To find $VaR_p(X)$, set

$$F_X(x) = p$$

$$\Leftrightarrow 1 - \frac{e^{[1-(1-e^{-\beta x})^\alpha]\cdot\lambda} - 1}{e^\lambda - 1} = p$$

$$\Leftrightarrow e^{[1-(1-e^{-\beta x})^\alpha]\cdot\lambda} - 1 = (1-p)(e^\lambda - 1)$$

$$\Leftrightarrow e^{[1-(1-e^{-\beta x})^\alpha]\cdot\lambda} = 1 + (1-p)(e^\lambda - 1)$$

$$\Leftrightarrow [1-(1-e^{-\beta x})^\alpha]\lambda = \log[1+(1-p)(e^\lambda - 1)]$$

$$\Leftrightarrow \quad 1 - \left(1-e^{-\beta x}\right)^{\alpha} = \frac{1}{\lambda} \log\left[1+(1-p)\left(e^{\lambda}-1\right)\right]$$

$$\Leftrightarrow \quad \left(1-e^{-\beta x}\right)^{\alpha} = 1-\frac{1}{\lambda} \log\left[1+(1-p)\left(e^{\lambda}-1\right)\right]$$

$$\Leftrightarrow \quad 1-e^{-\beta x} = \left\{1-\frac{1}{\lambda} \log\left[1+(1-p)\left(e^{\lambda}-1\right)\right\}^{\frac{1}{\alpha}}$$

$$\Leftrightarrow \quad VaR_{p}(X) = -\frac{1}{\beta} \log\left[1-\left\{1-\frac{1}{\lambda} \log\left[1+(1-p)\left(e^{\lambda}-1\right)\right\}^{\frac{1}{\alpha}}\right]$$

Q2, Exam 2016 / 2017 (c)

$$f(x) = C\, g(x) \left[G(x)\right]^{a-1} \left[1 - G(x)\right]^{b-1}$$

Show that F and G belong to the same domain of attraction.

i) Assume G belongs to Gumbel max domain and show that F also belongs to the Gumbel max domain

ii) Repeat for Fréchet max domain

iii) " " Weibull " " .

i) Assume G belongs to the Gumbel max domain. So there exists $\gamma(t) > 0$ such that

$$\lim_{t \to w(G)} \frac{1 - G(t + x\gamma(t))}{1 - G(t)} = e^{-x} \quad \ldots (*)$$

$$\lim_{t \to w(F)} \frac{1 - F(t + x\gamma(t))}{1 - F(t)}$$

$$\stackrel{LH}{=} \lim_{t \to w(F)} \frac{-f(t + x\gamma(t))\,(1 + x\gamma'(t))}{-f(t)}$$

$$= \lim_{t \to \omega(G)} \frac{f(t + x\gamma(t))\,(1 + x\gamma'(t))}{f(t)}$$

$$= \lim_{t \to \omega(G)} \frac{\cancel{C}\, g(t + x\gamma(t))}{\cancel{C}\, g(t)} \left[ \frac{\underbrace{G(t + x\gamma(t))}_{\to 1}}{\underbrace{G(t)}_{\to 1}} \right]^{a-1}$$

$$\cdot \left[ \frac{1 - G(t + x\gamma(t))}{1 - G(t)} \right]^{b-1} (1 + x\gamma'(t))$$

$$= \lim_{t \to \omega(G)} \frac{g(t + x\gamma(t))}{g(t)} \cdot (1 + x\gamma'(t))$$

$$\cdot \left[ \frac{1 - G(t + x\gamma(t))}{1 - G(t)} \right]^{b-1}$$

$$\overset{LH}{\underset{\text{in reverse}}{=}} \lim_{t \to \omega(G)} \frac{1 - G(t + x\gamma(t))}{1 - G(t)} \cdot \left[ \frac{1 - G(t + x\gamma(t))}{1 - G(t)} \right]^{b-1}$$

$$= \lim_{t \to \omega(G)} \left[ \frac{1 - G(t + x\gamma(t))}{1 - G(t)} \right]^{b}$$

$$\overset{\circledast}{=} \left[ e^{-x} \right]^{b} = e^{-bx}$$

Hence $F$ also belongs to the Gumbel max domain.