MATH3/4/68181: EXTREME VALUES AND FINANCIAL RISK
SEMESTER 1
QUIZ PROBLEM 5
(Deadline: Tuesday 5 December 2018, 12:00noon)

A random variable X is said to have the Student's t distribution if its probability density function is

$$f(x) = K\left(1 + \frac{x^2}{a}\right)^{-\frac{a+1}{2}}$$

for $-\infty < x < \infty$ and $a > 0$, where

$$K = \frac{\Gamma\left(\frac{a+1}{2}\right)}{\sqrt{a\pi}\Gamma\left(\frac{a}{2}\right)}.$$

Show that $\mathrm{ES}_p(X)$ is given by

$$\mathrm{ES}_p(X) = \frac{aK}{(1-a)p}\left\{1 + \frac{1}{a}\left[\mathrm{VaR}_p(X)\right]^2\right\}^{\frac{1-a}{2}}$$

provided that $a > 1$. Please give full details.

This problem is worth 2 marks. Marking scheme: 2 marks if the answer is correct, and the derivation is correct and detailed enough; 1 mark if the answer is correct, and the derivation is incorrect or not detailed enough; 1 mark if the answer is incorrect or not given, but the derivation is correct and detailed enough; 0 mark if the answer is correct, but the derivation is not detailed enough; 0 mark if the answer is incorrect, and the derivation is not detailed enough.

You can give your written solution to me during any of the lectures or example classes. You can also bring your solution to ATB2.223, place it under the door if I am not in. Email submissions or late submissions will not be accepted. I will mark your solutions and email your mark to you within 24 hours of the deadline. PLEASE DO NOT FORGET TO WRITE YOUR FULL NAME AND ID.