MATH48181/68181: EXTREME VALUES AND FINANCIAL RISK
SEMESTER 1
SOLUTIONS TO QUIZ PROBLEM 6

Suppose a portfolio is made of up of two independent investments. Let X and Y denote the losses. Assume that X and Y have the probability density functions

$$f_X(x) = a\exp(-ax)$$

and

$$f_Y(y) = b\exp(-by),$$

respectively, for $x > 0$, $y > 0$, $a > 0$ and $b > 0$.

Let $S = X + Y$. The probability density function of S is

$$\begin{aligned}
f_S(s) &= \int_0^s f_X(x) f_Y(s-x)dx \\
&= ab\int_0^s \exp(-ax)\exp\left[-b(s-x)\right]dx \\
&= ab\exp(-bs)\int_0^s \exp(bx-ax)dx \\
&= ab\exp(-bs)\left[\frac{\exp(bx-ax)}{b-a}\right]_0^s \\
&= \frac{ab}{b-a}\exp(-bs)\left[\exp(bs-as)-1\right] \\
&= \frac{ab}{b-a}\left[\exp(-as)-\exp(-bs)\right].
\end{aligned}$$

The cumulative distribution function of S is

$$\begin{aligned}
F_S(s) &= \int_0^s f_S(t)dt \\
&= \frac{ab}{b-a}\left[\int_0^s \exp(-at)dt - \int_0^s \exp(-bt)dt\right] \\
&= \frac{ab}{b-a}\left\{\left[\frac{\exp(-at)}{-a}\right]_0^s - \left[\frac{\exp(-bt)}{-b}\right]_0^s\right\} \\
&= \frac{ab}{b-a}\left\{\frac{1-\exp(-as)}{a} - \frac{1-\exp(-bs)}{b}\right\}.
\end{aligned}$$