MATH48181/68181: EXTREME VALUES AND FINANCIAL RISK
SEMESTER 1
SOLUTIONS TO QUIZ PROBLEM 5

Suppose a portfolio is made of up of k independent investments. Let $X_1, X_2, \ldots, X_k$ denote the losses. Assume that X_i are Uniform $[-\theta, \theta]$ random variables.

Let $V = \max(X_1, \ldots, X_k)$. The cumulative distribution function of V is

$$\begin{aligned} F_V(v) &= \Pr(V \leq v) \\ &= 1 - \Pr\left(\max\left(X_1, \ldots, X_k\right) \leq v\right) \\ &= \Pr\left(X_1 \leq v, \ldots, X_k \leq v\right) \\ &= \Pr\left(X_1 \leq v\right) \cdots \Pr\left(X_k \leq v\right) \\ &= \left[\Pr\left(X_1 \leq v\right)\right]^k \\ &= \left(\frac{v+\theta}{2\theta}\right)^k. \end{aligned}$$

So, the probability density function of V is

$$f_V(v) = \frac{k}{2\theta}\left(\frac{v+\theta}{2\theta}\right)^{k-1}$$

for $-\theta < v < \theta$. Hence, the expected maximum portfolio loss is

$$\begin{aligned} E(V) &= \frac{k}{2\theta}\int_{-\theta}^{\theta} v\left(\frac{v+\theta}{2\theta}\right)^{k-1} dv \\ &= \frac{k}{2\theta}\int_{-\theta}^{\theta} (v+\theta-\theta)\left(\frac{v+\theta}{2\theta}\right)^{k-1} dv \\ &= \frac{k}{(2\theta)^k}\int_{-\theta}^{\theta} (v+\theta-\theta)(v+\theta)^{k-1} dv \\ &= \frac{k}{(2\theta)^k}\left[\int_{-\theta}^{\theta} (v+\theta)^k dv - \theta\int_{-\theta}^{\theta} (v+\theta)^{k-1} dv\right] \\ &= \frac{k}{(2\theta)^k}\left\{\left[\frac{(v+\theta)^{k+1}}{k+1}\right]_{-\theta}^{\theta} - \theta\left[\frac{(v+\theta)^k}{k}\right]_{-\theta}^{\theta}\right\} \\ &= \frac{k}{(2\theta)^k}\left\{\frac{(2\theta)^{k+1}}{k+1} - \theta\frac{(2\theta)^k}{k}\right\} \\ &= k\theta\left\{\frac{2}{k+1} - \frac{1}{k}\right\} \\ &= \frac{\theta(k-1)}{k+1}. \end{aligned}$$