MATH48181/68181: EXTREME VALUES AND FINANCIAL RISK
SEMESTER 1
SOLUTIONS QUIZ PROBLEM 4

Suppose a portfolio is made of up of k independent investments. Let $X_1, X_2, \ldots, X_k$ denote the losses. Assume that X_i has the Uniform $[-\theta_i, \theta_i]$ distribution for $i = 1, 2, \ldots, k$.

Let $U = \min(X_1, X_2, \ldots, X_k)$. Its cumulative distribution function is

$$
\begin{aligned}
F_U(u) &= \Pr(U \leq u) \\
&= 1 - \Pr(U > u) \\
&= 1 - \Pr\left(\min\left(X_1, X_2, \ldots, X_k\right) > u\right) \\
&= 1 - \Pr\left(X_1 > u, X_2 > u, \ldots, X_k > u\right) \\
&= 1 - \Pr\left(X_1 > u\right) \Pr\left(X_2 > u\right) \cdots \Pr\left(X_k > u\right) \\
&= 1 - \left[1 - \Pr\left(X_1 \leq u\right)\right]\left[1 - \Pr\left(X_2 \leq u\right)\right] \cdots \left[1 - \Pr\left(X_k \leq u\right)\right] \\
&= 1 - \left[1 - \frac{u + \theta_1}{2\theta_1}\right]\left[1 - \frac{u + \theta_2}{2\theta_2}\right] \cdots \left[1 - \frac{u + \theta_k}{2\theta_k}\right] \\
&= 1 - \left[\frac{\theta_1 - u}{2\theta_1}\right]\left[\frac{\theta_2 - u}{2\theta_2}\right] \cdots \left[\frac{\theta_k - u}{2\theta_k}\right] \\
&= 1 - 2^{-k}\left(\prod_{i=1}^{k} \theta_i\right)^{-1}\left[\prod_{i=1}^{k}\left(\theta_i - u\right)\right].
\end{aligned}
$$

Let $V = \max(X_1, X_2, \ldots, X_k)$. Its cumulative distribution function is

$$
\begin{aligned}
F_V(v) &= \Pr(V \leq v) \\
&= \Pr\left(\max\left(X_1, X_2, \ldots, X_k\right) \leq v\right) \\
&= \Pr\left(X_1 \leq u, X_2 \leq u, \ldots, X_k \leq u\right) \\
&= \Pr\left(X_1 \leq u\right) \Pr\left(X_2 \leq u\right) \cdots \Pr\left(X_k \leq u\right) \\
&= \left[\frac{u + \theta_1}{2\theta_1}\right]\left[\frac{u + \theta_2}{2\theta_2}\right] \cdots \left[\frac{u + \theta_k}{2\theta_k}\right] \\
&= 2^{-k}\left(\prod_{i=1}^{k} \theta_i\right)^{-1}\left[\prod_{i=1}^{k}\left(u + \theta_i\right)\right].
\end{aligned}
$$