

MATH48181/68181: EXTREME VALUES AND FINANCIAL RISK
SEMESTER 1
SOLUTIONS TO QUIZ PROBLEM 2

Suppose X is a random variable with cumulative distribution function

$$F(x) = 1 - p^{(x+1)^2}$$

for $x = 0, 1, \dots$ and $0 < p < 1$.

Clearly, $w(F) = \infty$. Then,

$$\begin{aligned} \lim_{k \rightarrow \infty} \frac{p(k)}{1 - F(k-1)} &= \lim_{k \rightarrow \infty} \frac{F(k) - F(k-1)}{1 - F(k-1)} \\ &= \lim_{k \rightarrow \infty} \frac{1 - p^{(k+1)^2} - [1 - p^{k^2}]}{1 - [1 - p^{k^2}]} \\ &= \lim_{k \rightarrow \infty} \frac{p^{k^2} - p^{(k+1)^2}}{p^{k^2}} \\ &= \lim_{k \rightarrow \infty} 1 - p^{(k+1)^2 - k^2} \\ &= \lim_{k \rightarrow \infty} 1 - p^{k^2 + 2k + 1 - k^2} \\ &= \lim_{k \rightarrow \infty} 1 - p^{2k+1} \\ &= \lim_{k \rightarrow \infty} 1 - 0 \\ &= 1. \end{aligned}$$

Hence, there can be no domain of attraction that F can belong to.