MATH48181/68181: EXTREME VALUES AND FINANCIAL RISK
SEMESTER 1
SOLUTIONS TO QUIZ PROBLEM 1

Suppose X is a random variable with probability density function

$$f(x) = \sqrt{\frac{2}{\pi}} \exp\left(-\frac{x^2}{2}\right)$$

for $x > 0$.

Note that $w(F) = \infty$. Then

$$\begin{aligned}
\lim_{t\to\infty} \frac{1 - F(t + xg(t))}{1 - F(t)} &= \lim_{t\to\infty} \frac{-\left(1 + xg^{'}(t)\right) f(t + xg(t))}{-f(t)} \\
&= \lim_{t\to\infty} \frac{\left(1 + xg^{'}(t)\right) f(t + xg(t))}{f(t)} \\
&= \lim_{t\to\infty} \frac{\left(1 + xg^{'}(t)\right) \sqrt{\frac{2}{\pi}} \exp\left(-\frac{(t+xg(t))^2}{2}\right)}{\sqrt{\frac{2}{\pi}} \exp\left(-\frac{t^2}{2}\right)} \\
&= \lim_{t\to\infty} \frac{\left(1 + xg^{'}(t)\right) \exp\left(-\frac{(t+xg(t))^2}{2}\right)}{\exp\left(-\frac{t^2}{2}\right)} \\
&= \lim_{t\to\infty} \left(1 + xg^{'}(t)\right) \exp\left[\frac{t^2}{2} - \frac{(t + xg(t))^2}{2}\right] \\
&= \lim_{t\to\infty} \left(1 + xg^{'}(t)\right) \exp\left[\frac{t^2}{2} - \frac{t^2 + x^2 (g(t))^2 + 2txg(t)}{2}\right] \\
&= \lim_{t\to\infty} \left(1 + xg^{'}(t)\right) \exp\left[-\frac{x^2}{2} (g(t))^2 - txg(t)\right].
\end{aligned}$$

Now take $g(t) = 1/t$, then $g^{'}(t) = -1/t^2$ and

$$\lim_{t\to\infty} \left(1 - \frac{x}{t^2}\right) \exp\left[-\frac{x^2}{2}\frac{1}{t^2} - x\right] = \exp(-x).$$

Hence, the distribution belongs to the Gumbel domain of attraction.