MATH4/68181: EXTREME VALUES AND FINANCIAL RISK
SEMESTER 1
SOLUTION TO QUIZ PROBLEM 2

Suppose X is a random variable with probability mass function

$$p(k) = \Pr(X = k) = 2^{-k}$$

for $k = 1, 2, \ldots$. Using the fact that

$$\sum_{i=1}^{k} r^{i-1} = \frac{1 - r^k}{1 - r},$$

the corresponding cumulative distribution function can be calculated as

$$F(k) = \sum_{i=1}^{k} 2^{-i} = 2^{-1} \sum_{i=1}^{k} \left(2^{-1}\right)^{i-1} = 2^{-1} \frac{1 - \left(2^{-1}\right)^k}{1 - 2^{-1}} = 1 - 2^{-k}$$

for $k = 1, 2, \ldots$. So,

$$\frac{p(k)}{1 - F(k-1)} = \frac{2^{-k}}{1 - (1 - 2^{1-k})} = \frac{1}{2}.$$

Hence, the ETT does not hold for this distribution.