MATH20802: Statistical Methods
Semester 2
Formulas to remember for the final exam in May / June 2018

The moment generating function of a random variable X **is** $M_X(t) = E\left[\exp(tX)\right]$.

The fact that $E\left(X^n\right) = M_X^{('n)}(0)$.

$\widehat{\theta}$ **is an unbiased estimator of** θ **if** $E\left(\widehat{\theta}\right) = \theta$.

$\widehat{\theta}$ **is an asymptotically unbiased estimator of** θ **if** $\lim_{n\to\infty} E\left(\widehat{\theta}\right) = \theta$.

The bias of $\widehat{\theta}$ **is** $E\left(\widehat{\theta}\right) - \theta$.

The mean squared error of $\widehat{\theta}$ **is** $E\left[\left(\widehat{\theta} - \theta\right)^2\right]$.

$\widehat{\theta}$ **is a consistent estimator of** θ **if** $\lim_{n\to\infty} E\left[\left(\widehat{\theta} - \theta\right)^2\right] = 0$.

The Cauchy distribution has its probability density function specified by $f(x) = \frac{1}{\pi}\frac{1}{1+x^2}$ **for** $-\infty < x < +\infty$.

The gamma function is defined by $\Gamma(a) = \int_0^\infty t^{a-1}\exp(-t)dt$.

The fact that $\Gamma(n) = (n-1)!$.

The fact that $\Gamma(x+1) = x\Gamma(x)$.

The beta function is defined by $B\left(a,b\right) = \int_0^1 t^{a-1}(1-t)^{b-1}dt$.

The fact that $B\left(a,b\right) = \dfrac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$.

The probability density function of $X \sim Exp\left(\lambda\right)$ **is** $f_X(x) = \lambda\exp\left(-\lambda x\right)$.

The cumulative distribution function of $X \sim Exp\left(\lambda\right)$ **is** $F_X(x) = 1 - \exp\left(-\lambda x\right)$.

The probability density function of $X \sim Uni\left(a,b\right)$ **is** $f_X(x) = \dfrac{1}{b-a}$.

The probability density function of an inverse Gaussian random variable is

$$f_X(x) = \sqrt{\frac{\lambda}{2\pi x^3}}\exp\left[-\frac{\lambda(x-\mu)^2}{2\mu^2 x}\right]$$

for $x > 0$, $\mu > 0$ **and** $\lambda > 0$. **The mean and variance are** $E(X) = \mu$ **and** $Var(X) = \frac{\mu^3}{\lambda}$.

The mean and variance of $X \sim \chi^2_\nu$ **are** $E(X) = \nu$ **and** $Var(X) = 2\nu$.

If $X_1, X_2, \ldots, X_n$ **is a random sample from** $N\left(\mu, \sigma^2\right)$ **then** $\frac{1}{\sigma^2}\sum_{i=1}^n \left(X_i - \overline{X}\right)^2 \sim \chi^2_{n-1}$, **where** $\overline{X}$ **denotes the sample mean.**

The Type I error of $H_0 : \mu = \mu_0$ **versus** $H_1 : \mu \neq \mu_0$ **occurs if** H_0 **is rejected when in fact** $\mu = \mu_0$.

The Type II error of $H_0 : \mu = \mu_0$ **versus** $H_1 : \mu \neq \mu_0$ **occurs if** H_0 **is accepted when in fact** $\mu \neq \mu_0$.

The significance level of $H_0 : \mu = \mu_0$ versus $H_1 : \mu \neq \mu_0$ is the probability of type I error.

The power function of $H_0 : \mu = \mu_0$ versus $H_1 : \mu \neq \mu_0$ is $\Pi(\mu) = \Pr\left(\text{Reject} H_0 \mid \mu\right)$.

Suppose $X_1, X_2, \ldots, X_n$ is a random sample from $N\left(\theta, \sigma^2\right)$, where σ is not known.

The rejection region for $H_0 : \sigma = \sigma_0$ versus $H_1 : \sigma \neq \sigma_0$ is

$$\frac{(n-1)S^2}{\sigma_0^2} < \chi^2_{n-1,1-\alpha/2} \textbf{ or } \frac{(n-1)S^2}{\sigma_0^2} > \chi^2_{n-1,\alpha/2}.$$

The rejection region for $H_0 : \sigma = \sigma_0$ versus $H_1 : \sigma < \sigma_0$ is

$$\frac{(n-1)S^2}{\sigma_0^2} < \chi^2_{n-1,1-\alpha}.$$

The rejection region for $H_0 : \sigma = \sigma_0$ versus $H_1 : \sigma > \sigma_0$ is

$$\frac{(n-1)S^2}{\sigma_0^2} > \chi^2_{n-1,\alpha}.$$

The method of mathematical induction.

Based on a random sample $X_1, X_2, \ldots, X_n$ from a distribution with the probability density function $f(x;\theta)$, the Neyman-Pearson test rejects $H_0 : \theta = \theta_1$ versus $H_1 : \theta = \theta_2$ if

$$\frac{L\left(\theta_1\right)}{L\left(\theta_2\right)} = \frac{\prod_{i=1}^{n} f\left(X_i;\theta_1\right)}{\prod_{i=1}^{n} f\left(X_i;\theta_2\right)} < k$$

for some k.