MATH4/68181: Extreme values and financial risk
Semester 1
Problem sheet 5

1. If $X_i \sim Exp(\lambda_i)$, $i = 1, 2$ are independent random variables then find $\Pr(X_1 < X_2)$.

2. If $X_i \sim Exp(\lambda_i)$, $i = 1, 2, 3$ are independent random variables then find $\Pr(X_1 < X_2 < X_3)$.

3. If $X_i \sim Exp(\lambda_i)$, $i = 1, 2, 3, 4$ are independent random variables then find $\Pr(X_1 < X_2 < X_3 < X_4)$.

4. If $X_i \sim Exp(\lambda_i)$, $i = 1, 2, \ldots, k$ are independent random variables then find the general formula for $\Pr(X_1 < X_2 < \cdots < X_k)$.

5. If $X_i \sim N(\mu_i, \sigma_i^2)$, $i = 1, 2$ are independent random variables then find $\Pr(X_1 < X_2)$.

6. Suppose X_i, $i = 1, 2$ are independent Pareto random variables with cdfs specified by

$$F_{X_1}(x) = 1 - (K/x)^a, \ x \geq K$$

and

$$F_{X_2}(y) = 1 - (L/y)^b, \ y \geq L,$$

respectively, where $K > 0$, $L > 0$, $a > 0$ and $b > 0$. Find $\Pr(X_1 < X_2)$.

7. Suppose X_i, $i = 1, 2$ are independent Rayleigh random variables with cdfs specified by

$$F_{X_1}(x) = 1 - \exp\left(-\frac{x^2}{2\sigma_1^2}\right), \ x \geq 0$$

and

$$F_{X_2}(y) = 1 - \exp\left(-\frac{y^2}{2\sigma_2^2}\right), \ y \geq 0,$$

respectively, where $\sigma_1 > 0$ and $\sigma_2 > 0$. Find $\Pr(X_1 < X_2)$.

8. If $X_i \sim uniform[a_i, b_i]$, $i = 1, 2$ are independent uniform random variables then find $\Pr(X_1 < X_2)$.