MATH4/68181: Extreme values and financial risk
Semester 1
Problem sheet for Week 3

1. Suppose $X_1, X_2, \ldots, X_n$ is a random sample from Bernoulli (p) and let $M_n = \max(X_1, X_2, \ldots, X_n)$. Determine if $(M_n - b_n)/a_n$ for suitable norming constants a_n and b_n will have a non-degenerate limiting distribution as $n \to \infty$.

2. Suppose $X_1, X_2, \ldots, X_n$ is a random sample from the degenerate distribution specified by the pmf

$$p(k) = \begin{cases} 1, & \text{if } k = k_0, \\ 0, & \text{if } k \neq k_0. \end{cases}$$

   Determine if $(M_n - b_n)/a_n$ for suitable norming constants a_n and b_n will have a non-degenerate limiting distribution as $n \to \infty$.

3. Suppose $X_1, X_2, \ldots, X_n$ is a random sample from the Yule distribution specified by the pmf

$$p(k) = \rho B(k, \rho + 1), \ k \geq 1.$$

   Determine if $(M_n - b_n)/a_n$ for suitable norming constants a_n and b_n will have a non-degenerate limiting distribution as $n \to \infty$.

4. Suppose $X_1, X_2, \ldots, X_n$ is a random sample from the zeta distribution specified by the pmf

$$p(k) = k^{-s}/\zeta(s), \ k \geq 1,$$

   where $\zeta(\cdot)$ denotes the Riemann zeta function. Determine if $(M_n - b_n)/a_n$ for suitable norming constants a_n and b_n will have a non-degenerate limiting distribution as $n \to \infty$.

5. Suppose $X_1, X_2, \ldots, X_n$ is a random sample from the Gauss-Kuzmin distribution specified by the pmf

$$p(k) = -\log_2 \left[1 - (k+1)^{-2}\right], \ k \geq 1,$$

   and the cdf

$$F(k) = 1 - \log_2 \left[\frac{k+2}{k+1}\right], \ k \geq 1.$$

   Determine if $(M_n - b_n)/a_n$ for suitable norming constants a_n and b_n will have a non-degenerate limiting distribution as $n \to \infty$.

6. Suppose $X_1, X_2, \ldots, X_n$ is a random sample from the discrete Lindley distribution specified by the pmf

$$p(x) = \frac{p^x}{1+\theta} \left\{\theta(1-2p) + (1-p)(1+\theta x)\right\},$$

   where $p = \exp(-\theta)$, for $\theta > 0$ and $x = 0, 1, 2, \ldots$. Determine if $(M_n - b_n)/a_n$ for suitable norming constants a_n and b_n will have a non-degenerate limiting distribution as $n \to \infty$.

7. Suppose $X_1, X_2, \ldots, X_n$ is a random sample from the discrete Weibull distribution specified by the cdf

$$F(x) = 1 - q^{(x+1)^a},$$

where $0 < q < 1$, for $a > 0$ and $x = 0, 1, 2, \ldots$. Determine if $(M_n - b_n)/a_n$ for suitable norming constants a_n and b_n will have a non-degenerate limiting distribution as $n \to \infty$.