MATH4/68181: Extreme values and financial risk
Semester 1
Problem sheet for Week 9

Show each of the following is a valid copula.

1. the independence copula defined by $C(u_1, u_2) = u_1 u_2$.
2. the copula defined by $C(u_1, u_2) = \min(u_1, u_2)$.
3. the copula defined by

$$C(u_1, u_2) = u_1 u_2 \exp[-\theta \log u_1 \log u_2]$$

for $0 < \theta \leq 1$.

4. the Farlie-Gumbel-Morgenstern copula defined by

$$C(u_1, u_2) = u_1 u_2 [1 + \phi (1 - u_1)(1 - u_2)]$$

for $-1 \leq \phi \leq 1$.

5. Burr copula defined by

$$C(u_1, u_2) = u_1 + u_2 - 1 + \left[(1 - u_1)^{-1/\alpha} + (1 - u_2)^{-1/\alpha} - 1\right]^{-\alpha}$$

for $\alpha > 0$.

6. Marshall and Olkin's copula defined by

$$C(u_1, u_2) = \begin{cases} u_1^{1-\alpha} u_2, & \text{if } u_1^\alpha \geq u_2^\beta, \\ \\ u_1 u_2^{1-\beta}, & \text{if } u_1^\alpha < u_2^\beta \end{cases}$$

for $0 \leq \alpha, \beta \leq 1$.