

MATH4/68181: Extreme values and financial risk
Semester 1
Problem sheet 13

1) For the ARCH (q) model given by

$$e_t = \sigma_t Z_t$$

and

$$\sigma_t^2 = \alpha_0 + \alpha_1 e_{t-1}^2 + \cdots + \alpha_q e_{t-q}^2,$$

find the mean and variance of e_t , assuming $Z_t \sim N(0, 1)$.

2) For the GARCH (p, q) model given by

$$e_t = \sigma_t Z_t$$

and

$$\sigma_t^2 = \alpha_0 + \alpha_1 e_{t-1}^2 + \cdots + \alpha_q e_{t-q}^2 + \beta_1 \sigma_{t-1}^2 + \cdots + \beta_p \sigma_{t-p}^2,$$

find the mean and variance of e_t , assuming $Z_t \sim N(0, 1)$.

3) For the NGARCH model given by

$$e_t = \sigma_t Z_t$$

and

$$\sigma_t^2 = \omega + \alpha (e_{t-1} - \theta \sigma_{t-1})^2 + \beta \sigma_{t-1}^2,$$

find the mean and variance of e_t , assuming $Z_t \sim N(0, 1)$.

4) For the QGARCH model given by

$$e_t = \sigma_t Z_t$$

and

$$\sigma_t^2 = K + \alpha e_{t-1}^2 + \beta \sigma_{t-1}^2 + \phi e_{t-1},$$

find the mean and variance of e_t , assuming $Z_t \sim N(0, 1)$.

5) For the GJR-QGARCH model given by

$$e_t = \sigma_t Z_t,$$

$$\sigma_t^2 = K + \delta \sigma_{t-1}^2 + \alpha e_{t-1}^2 + \phi e_{t-1}^2 I_{t-1},$$

and

$$I_{t-1} = \begin{cases} 0, & \text{if } e_{t-1} \geq 0, \\ 1, & \text{if } e_{t-1} < 0, \end{cases}$$

find the mean and variance of e_t , assuming $Z_t \sim N(0, 1)$.