MATH4/68181: Extreme values and financial risk
Semester 1
Problem sheet for Week 10

1) Consider a bivariate distribution specified by the joint survival function

$$\overline{F}(x,y) = \exp\left[-\frac{\theta y^2}{x+y} + \theta y - x - y\right]$$

for $x > 0$ and $y > 0$.

(a) show that the distribution is a bivariate extreme value distribution;

(b) derive the joint cdf;

(c) derive the conditional cdf if Y given $X = x$;

(d) derive the conditional cdf if X given $Y = y$;

(e) derive the joint pdf;

(f) derive the conditional pdf of Y given $X = x$;

(g) derive the conditional pdf of X given $Y = y$.

2) Consider a bivariate distribution specified by the joint survival function

$$\overline{F}(x,y) = \exp\left[\frac{\alpha xy}{x+y} - x - y\right]$$

for $x > 0$ and $y > 0$.

(a) show that the distribution is a bivariate extreme value distribution;

(b) derive the joint cdf;

(c) derive the conditional cdf if Y given $X = x$;

(d) derive the conditional cdf if X given $Y = y$;

(e) derive the joint pdf;

(f) derive the conditional pdf of Y given $X = x$;

(g) derive the conditional pdf of X given $Y = y$.

3) Consider a bivariate distribution specified by the joint survival function

$$\overline{F}(x,y) = \exp\left[-\left(x^a + y^a\right)^{1/a}\right]$$

for $x > 0$ and $y > 0$.

(a) show that the distribution is a bivariate extreme value distribution;

(b) derive the joint cdf;

(c) derive the conditional cdf if Y given $X = x$;

(d) derive the conditional cdf if X given $Y = y$;

(e) derive the joint pdf;

(f) derive the conditional pdf of Y given $X = x$;

(g) derive the conditional pdf of X given $Y = y$.