MATH4/68181: Extreme values and financial risk
Semester 1
Problem sheet for Week 2

1. Find the density functions of $\Lambda(x)$, $\Phi_\alpha(x)$, and $\Psi_\alpha(x)$.
2. Find the means corresponding to $\Lambda(x)$, $\Phi_\alpha(x)$, and $\Psi_\alpha(x)$.
3. Find the variances corresponding to $\Lambda(x)$, $\Phi_\alpha(x)$, and $\Psi_\alpha(x)$.
4. Show that $\Lambda^n(x) = \Lambda\left(\alpha_n x + \beta_n\right)$ if and only if $\alpha_n = 1$ and $\beta_n = -\log n$.
5. Show that $\Phi_\alpha^n(x) = \Phi_\alpha\left(\alpha_n x + \beta_n\right)$ if and only if $\alpha_n = n^{-1/\alpha}$ and $\beta_n = 0$.
6. Show that $\Psi_\alpha^n(x) = \Psi_\alpha\left(\alpha_n x + \beta_n\right)$ if and only if $\alpha_n = n^{1/\alpha}$ and $\beta_n = 0$.
7. Find the max domain of attraction of the exponential cdf $F(x) = 1 - \exp(-x)$.
8. Find the max domain of attraction of the exponentiated exponential cdf $F(x) = [1-\exp(-x)]^\alpha$.
9. Find the max domain of attraction of the uniform$[0,1]$ cdf $F(x) = x$.
10. Find the max domain of attraction of the Pareto cdf $F(x) = 1 - (K/x)^\alpha$.
11. Consider a class of distributions defined by the cdf

$$F(x) = K \int_0^{G(x)} t^{a-1}(1-t)^{b-1} \exp(-ct) dt,$$

and the pdf

$$f(x) = K g(x) G(x)^{a-1} \left\{1 - G(x)\right\}^{b-1} \exp\left\{-c\, G(x)\right\},$$

where $a > 0$, $b > 0$, $-\infty < c < \infty$, $G(\cdot)$ is a valid cdf and $g(x) = dG(x)/dx$. Show that F belongs to the same max domain of attraction as G. You may assume $w(F) = w(G)$.

12. Consider a class of distributions defined by the cdf

$$F(x) = \frac{\beta^a}{B(a,b)} \int_{-\infty}^{x} \frac{g(t)\left[G(t)\right]^{a-1}\left[1-G(t)\right]^{b-1}}{\left[1-(1-\beta)G(t)\right]^{a+b}} dt$$

and the pdf

$$f(x) = \frac{\beta^a}{B(a,b)} \frac{g(x)\left[G(x)\right]^{a-1}\left[1-G(x)\right]^{b-1}}{\left[1-(1-\beta)G(x)\right]^{a+b}}.$$

where $a > 0$, $b > 0$, $\beta > 0$, $G(\cdot)$ is a valid cdf and $g(x) = dG(x)/dx$. Show that F belongs to the same max domain of attraction as G. You may assume $w(F) = w(G)$.

13. If

$$G(x) = \exp\left\{-\left(1 + \xi\frac{x-\mu}{\sigma}\right)^{-1/\xi}\right\},$$

the GEV cdf, show that

$$G^{-1}(p) = \mu - \frac{\sigma}{\xi}\left[1 - \left\{-\log p\right\}^{-\xi}\right].$$

14. If

$$G(x) = 1 - \left\{1 + \xi \frac{x - t}{\sigma}\right\}^{-1/\xi},$$

the GP cdf, show that

$$G^{-1}(p) = t + \frac{\sigma}{\xi}\left\{(1-p)^{-\xi} - 1\right\}.$$