MATH48181/68181: Extreme values and financial risk
Semester 1
Formulas to remember for in-class test on Friday 17 December 2021

Extremal type theorem: Suppose $X_1, X_2, \ldots$ are independent and identically distributed (iid) random variables with common cumulative distribution function F. Let $M_n = \max\{X_1, \ldots, X_n\}$ denote the maximum of the first n random variables and let $w(F) = \sup\{x : F(x) < 1\}$ denote the upper end point of F. If there are norming constants $a_n > 0$, b_n and a nondegenerate G such that the cumulative distribution function of a normalized version of M_n converges to G, i.e.

$$\Pr\left(\frac{M_n - b_n}{a_n} \leq x\right) = F^n\left(a_n x + b_n\right) \to G(x)$$

as $n \to \infty$ then G must be of the same type as (cumulative distribution functions G and G^* are of the same type if $G^*(x) = G(ax + b)$ for some $a > 0$, b and all x) as one of the following three classes:

$$\begin{aligned}
I &: \Lambda(x) = \exp\left\{-\exp(-x)\right\}, \qquad x \in \Re; \\
II &: \Phi_\alpha(x) = \begin{cases} 0 & \textbf{if } x < 0, \\ \exp\left\{-x^{-\alpha}\right\} & \textbf{if } x \geq 0 \end{cases} \\
&\quad \textbf{for some } \alpha > 0; \\
III &: \Psi_\alpha(x) = \begin{cases} \exp\left\{-(-x)^{\alpha}\right\} & \textbf{if } x < 0, \\ 1 & \textbf{if } x \geq 0 \end{cases} \\
&\quad \textbf{for some } \alpha > 0.
\end{aligned}$$

Necessary and sufficient conditions for the three extreme value distributions:

$$\begin{aligned}
I &: \exists \gamma(t) > 0 \textbf{ s.t. } \lim_{t \uparrow w(F)} \frac{1 - F\left(t + x\gamma(t)\right)}{1 - F(t)} = \exp(-x), \qquad x > 0, \\
II &: w(F) = \infty \textbf{ and } \lim_{t \uparrow \infty} \frac{1 - F(tx)}{1 - F(t)} = x^{-\alpha}, \qquad x > 0, \\
III &: w(F) < \infty \textbf{ and } \lim_{t \downarrow 0} \frac{1 - F\left(w(F) - tx\right)}{1 - F\left(w(F) - t\right)} = x^{\alpha}, \qquad x > 0.
\end{aligned}$$