# MATH48181: Extreme values and financial risk
# Semester 1
# Formulas to remember for in-class test on 13 November 2018

**Extremal type theorem: Suppose $X_1, X_2, \ldots$ are independent and identically distributed (iid) random variables with common cumulative distribution function (cdf) $F$. Let $M_n = \max\{X_1, \ldots, X_n\}$ denote the maximum of the first $n$ random variables and let $w(F) = \sup\{x : F(x) < 1\}$ denote the upper end point of $F$. If there are norming constants $a_n > 0$, $b_n$ and a nondegenerate $G$ such that the cdf of a normalized version of $M_n$ converges to $G$, i.e.**

$$\Pr\left(\frac{M_n - b_n}{a_n} \le x\right) = F^n\left(a_n x + b_n\right) \to G(x)$$

**as $n \to \infty$ then $G$ must be of the same type as (cdfs $G$ and $G^*$ are of the same type if $G^*(x) = G(ax+b)$ for some $a > 0$, $b$ and all $x$) as one of the following three classes:**

$$\begin{aligned}
I &: \Lambda(x) = \exp\left\{-\exp(-x)\right\}, \qquad x \in \Re; \\
II &: \Phi_\alpha(x) = \begin{cases} 0 & \textbf{if } x < 0, \\ \exp\left\{-x^{-\alpha}\right\} & \textbf{if } x \ge 0 \end{cases} \\
&\quad \textbf{for some } \alpha > 0; \\
III &: \Psi_\alpha(x) = \begin{cases} \exp\left\{-(-x)^{\alpha}\right\} & \textbf{if } x < 0, \\ 1 & \textbf{if } x \ge 0 \end{cases} \\
&\quad \textbf{for some } \alpha > 0.
\end{aligned}$$

**Necessary and sufficient conditions for the three extreme value distributions:**

$$\begin{aligned}
I &: \exists \gamma(t) > 0 \textbf{ s.t. } \lim_{t \uparrow w(F)} \frac{1 - F\left(t + x\gamma(t)\right)}{1 - F(t)} = \exp(-x), \qquad x \in \Re, \\
II &: w(F) = \infty \textbf{ and } \lim_{t \uparrow \infty} \frac{1 - F(tx)}{1 - F(t)} = x^{-\alpha}, \qquad x > 0, \\
III &: w(F) < \infty \textbf{ and } \lim_{t \downarrow 0} \frac{1 - F\left(w(F) - tx\right)}{1 - F\left(w(F) - t\right)} = x^{\alpha}, \qquad x > 0.
\end{aligned}$$

**L'Hôpital's rule:**

$$\lim_{x \to c} \frac{f_1(x)}{f_2(x)} = \lim_{x \to c} \frac{f_1^{'}(x)}{f_2^{'}(x)}$$

**if $\lim_{x \to c} f_1(x) = \lim_{x \to c} f_2(x) = 0$ or $\pm\infty$.**

**The fact that: $(1-x)^a \approx 1 - ax$ for $x$ close to zero.**

**The fact that: $\exp(-x) \approx 1 - x$ for $x$ close to zero.**