

**MATH48181/68181: EXTREME VALUES**  
**FIRST SEMESTER**  
**IN CLASS TEST - 17 DECEMBER 2021**

**QUESTION** Suppose  $X_1, X_2, \dots, X_n$  is a random sample with cumulative distribution function  $F(\cdot)$ . State the Extremal Types Theorem for  $M_n = \max(X_1, X_2, \dots, X_n)$ . You must clearly specify the cumulative distribution functions of each of the three extreme value distributions. (1 marks)

State in full the necessary and sufficient conditions for  $F(\cdot)$  to belong to the domain of attraction of each of the three extreme value distributions. (1 marks)

Consider a class of distributions defined by the cumulative distribution function

$$F(x) = 1 - [1 - G(x)]^{G(x)}$$

where  $G(\cdot)$  is a valid cumulative distribution function. Show that  $F$  belongs to the same max domain of attraction as  $G$ . You may assume that  $F$  and  $G$  have the same upper end points. (8 marks)