MATH48181/68181: EXTREME VALUES FIRST SEMESTER IN CLASS TEST - 15 DECEMBER 2020

QUESTION Suppose $X_1, X_2, ..., X_n$ is a random sample with cumulative distribution function $F(\cdot)$. State the Extremal Types Theorem for $M_n = \max(X_1, X_2, ..., X_n)$. You must clearly specify the cumulative distribution functions of each of the three extreme value distributions. (1 marks)

State in full the necessary and sufficient conditions for $F(\cdot)$ to belong to the domain of attraction of each of the three extreme value distributions. (1 marks)

Consider a class of distributions defined by the cumulative distribution function

$$F(x) = \frac{G(x)}{a + (1 - a)G(x)},$$

where a > 0 and $G(\cdot)$ is a valid cumulative distribution function. Show that F belongs to the same max domain of attraction as G. You may assume that F and G have the same upper end points. (8 marks)