MATH48181: EXTREME VALUES
FIRST SEMESTER
IN CLASS TEST - 14 NOVEMBER 2017

YOUR FULL NAME:

YOUR ID:

This test contains three questions. Please answer ALL of the questions. You must fully explain all your answers. This test will account for 20 percent of your final mark.

Each paper will be graded by myself. If you would have complaints about your mark please address them directly to me.

Good luck.

PLEASE DO NOT TURN OVER UNTIL I SAY SO

FOR OFFICE USE ONLY

Q1	Q2	Q3	Total

QUESTION 1 i) Suppose $X_1, X_2, \ldots, X_n$ is a random sample with cdf $F(\cdot)$. State the Extremal Types Theorem for $M_n = \max(X_1, X_2, \ldots, X_n)$. You must clearly specify the cdfs of each of the three extreme value distributions. (3 marks)

ii) State in full the necessary and sufficient conditions for $F(\cdot)$ to belong to the domain of attraction of each of the three extreme value distributions. (3 marks)

iii) Consider a class of distributions defined by the cdf

$$F(x) = \frac{1}{2} \int_{-\infty}^{x} \{-\log[1 - G(y)]\}^2 g(y) dy$$

and the pdf

$$f(x) = \frac{1}{2} \{-\log[1 - G(x)]\}^2 g(x),$$

where $G(\cdot)$ is a valid cdf, and $g(x) = dG(x)/dx$. Show that F belongs to the same max domain of attraction as G. You may assume that F and G have the same upper end points. (4 marks)

QUESTION 2 Determine the domain of attraction (if there is one) for each of the following distributions (please give full details of your working):

(i) The exponentiated exponential distribution given by the cdf

$$F(x) = [1 - \exp(-x)]^2$$

for $x > 0$. (2 marks)

(ii) The exponentiated exponential geometric distribution given by the cdf

$$F(x) = \frac{\{1 - \exp(-2x)\}^2}{0.5 + 0.5\{1 - \exp(-2x)\}^2}$$

for $x > 0$. (2 marks)

(iii) The geometric distribution with the pmf and cdf specified by

$$p(k) = p(1-p)^{k-1}$$

and

$$F(k) = 1 - (1-p)^k,$$

where $0 < p < 1$ and $k = 1, 2, \ldots$; (2 marks)

(iv) The distribution given by the cdf

$$F(x) = \Phi^2(x), -\infty < x < \infty,$$

where

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} \exp\left(-\frac{y^2}{2}\right) dy$$

denotes the cdf of a standard normal random variable; (2 marks)

(v) The Gumbel distribution given by the cdf

$$F(x) = \exp\{-\exp(-x)\}, -\infty < x < \infty.$$

(2 marks)

QUESTION 3 Suppose a portfolio is made up of α investments where α is known. Suppose also that the losses on the investments say X_i, $i = 1, 2, \ldots, \alpha$ are independent and identical Pareto random variables specified by the cdf $F(x) = 1 - (K/x)^a$, $x \geq K$, where both $a > 0$ and $K > 0$ are unknown parameters. Let $Y = \max(X_1, \ldots, X_\alpha)$. Do the following:

(i) Find the cdf of Y; (2 marks)

(ii) Find the pdf of Y; (2 marks)

(iii) Find the mean of Y; (3 marks)

(iv) Find the variance of Y. (3 marks)