MATH38181: EXTREME VALUES
FIRST SEMESTER
IN CLASS TEST - 11 NOVEMBER 2014

YOUR FULL NAME:

YOUR ID:

This test contains two questions. Please answer ALL of the questions. You must fully explain all your answers. This test will account for 20 percent of your final mark.

Each paper will be graded by myself. If you would have complaints about your mark please address them directly to me.

Good luck.

PLEASE DO NOT TURN OVER UNTIL I SAY SO

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Q1	Q2	Total

QUESTION 1 Suppose $X_1, X_2, \ldots, X_n$ is a random sample with cdf $F(\cdot)$. State the Extremal Types Theorem for $M_n = \max(X_1, X_2, \ldots, X_n)$. You must clearly specify the cdfs of each of the three extreme value distributions. (3 marks)

State in full the necessary and sufficient conditions for $F(\cdot)$ to belong to the domain of attraction of each of the three extreme value distributions. (3 marks)

Consider a class of distributions defined by the cdf

$$F(x) = \frac{1}{2} \int_{-\infty}^{x} \left\{ -\log[1 - G(y)] \right\}^2 g(y) dy$$

and the pdf

$$f(x) = \frac{1}{2} \left\{ -\log[1 - G(x)] \right\}^2 g(x),$$

where $G(\cdot)$ is a valid cdf, and $g(x) = dG(x)/dx$. Show that F belongs to the same max domain of attraction as G. You may assume that F and G have the same upper end points. (4 marks)

QUESTION 2 State the domain of attraction (if there is one) for each of the following distributions:

(i) The exponentiated exponential distribution given by the cdf

$$F(x) = [1 - \exp(-x)]^2$$

for $x > 0$. (2 marks)

(ii) The exponentiated exponential geometric distribution given by the cdf

$$F(x) = \frac{\{1 - \exp(-2x)\}^2}{0.5 + 0.5\{1 - \exp(-2x)\}^2}$$

for $x > 0$. (2 marks)

(iii) The exponential-negative binomial distribution given by the cdf

$$F(x) = 1 - \frac{(1 - 0.5)^2 \exp(-4x)}{[1 - 0.5 \exp(-2x)]^2}$$

for $x > 0$. (2 marks)

(iv) The degenerate distribution given by the pmf

$$p(x) = \begin{cases} 1, & \text{if } x = 2, \\ 0, & \text{if } x \neq 2. \end{cases}$$

(2 marks)

(v) The Poisson distribution given by the pmf

$$p(x) = \frac{2^x \exp(-2)}{x!}$$

for $x = 0, 1, \ldots$. (2 marks)