Underreported/Overreported Income

In the economic literature, the under reported income is commonly expressed by the multiplicative relationship Z = XY, where Y is a multiplicative error and X denotes the true income. It is known that if Y has the power function distribution then X is Pareto distributed if and only if Z is also, see Krishnaji (1970).

The over reported income is commonly expressed by the multiplicative relationship Z = X/Y, where X and Y are independent random variables with X denoting the true income and Y a multiplicative error taking values in the interval (0, 1). It is known that if Y has the power function distribution then X is Pareto distributed if and only if Z is also, see Krishnaji (1970).

A Pareto random variable has cdf specified by $F(x) = 1 - (K/x)^a$ for x > K. A power function random variable has cdf specified by $F(x) = x^c$ for 0 < x < 1.

References

 Krishnaji, N. (1970). Characterization of the Pareto distribution through a model of underreported incomes. *Econometrica*, 38, 251-255.