c) Semi-parametric estimation methods for VaR

1) GEV method

Suppose the data follow definition 1 of extreme values. Then the CDF

$$F(x) = e^{-\left(1+\xi\frac{x-\mu}{\sigma}\right)^{-\frac{1}{\xi}}}.$$

Set $F(x) = p$

$$\Leftrightarrow \quad e^{-\left(1+\xi\frac{x-\mu}{\sigma}\right)^{-\frac{1}{\xi}}} = p$$

$$\Leftrightarrow \quad x = \mu + \frac{\sigma}{\xi}\left[(-\log p)^{-\xi} - 1\right]$$

$$\Leftrightarrow \quad VaR_p = \mu + \frac{\sigma}{\xi}\left[(-\log p)^{-\xi} - 1\right]$$

i) estimate ξ as

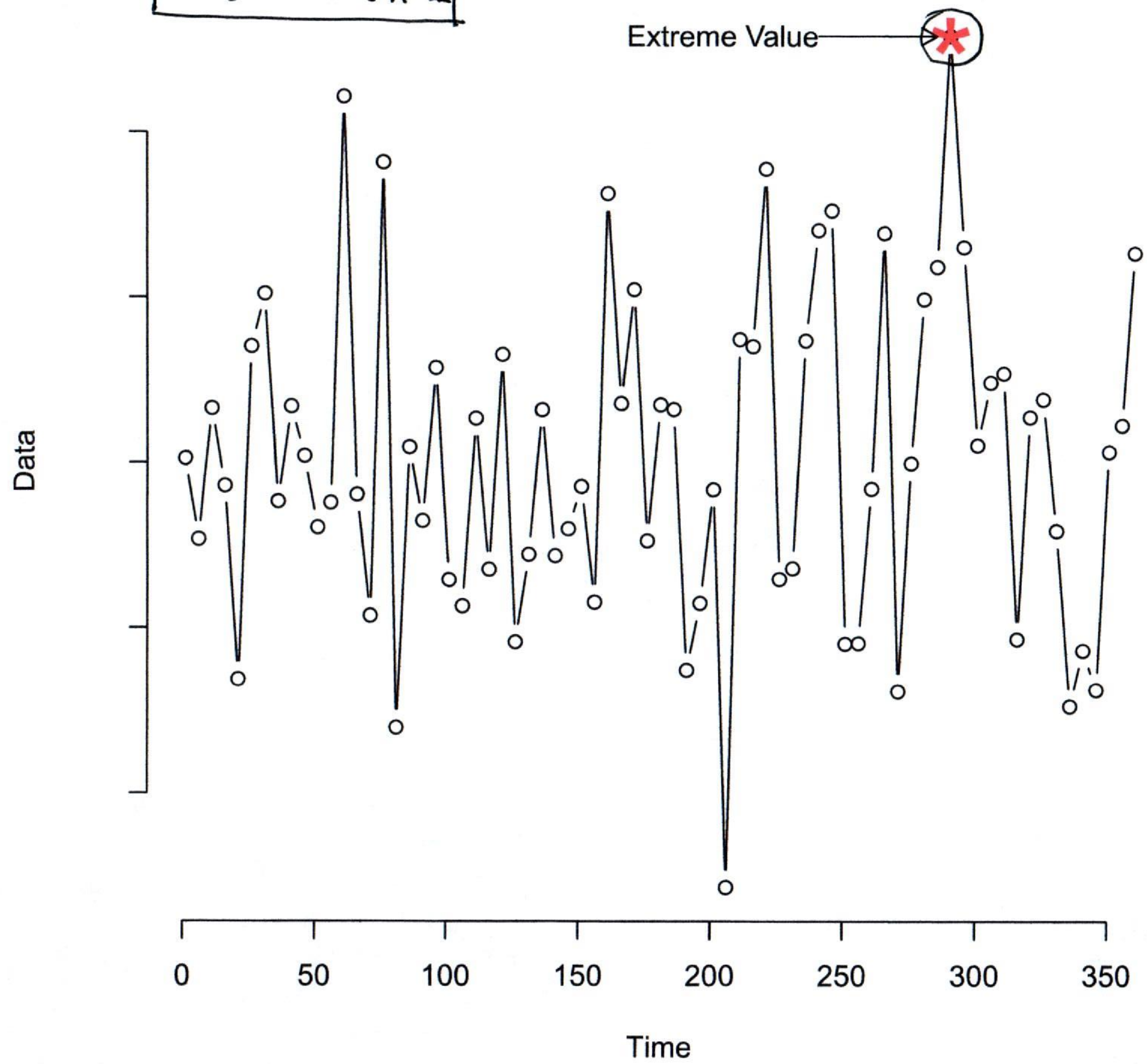
$$\boxed{\hat{\xi} = \frac{1}{k}\sum_{i=1}^{k} \log\frac{x_{(i)}}{x_{(k+1)}}, \quad 1 \leq k \leq n}$$

Hill's estimator

where $x_{(1)} \geq x_{(2)} \geq \cdots \geq x_{(n)}$ are data in decreasing order

$\hat{\xi}$ = a non-parametric estimator

Definition 1



ii) take $\hat{\mu}$ & $\hat{\sigma}$ as the MLEs of μ & σ, respectively

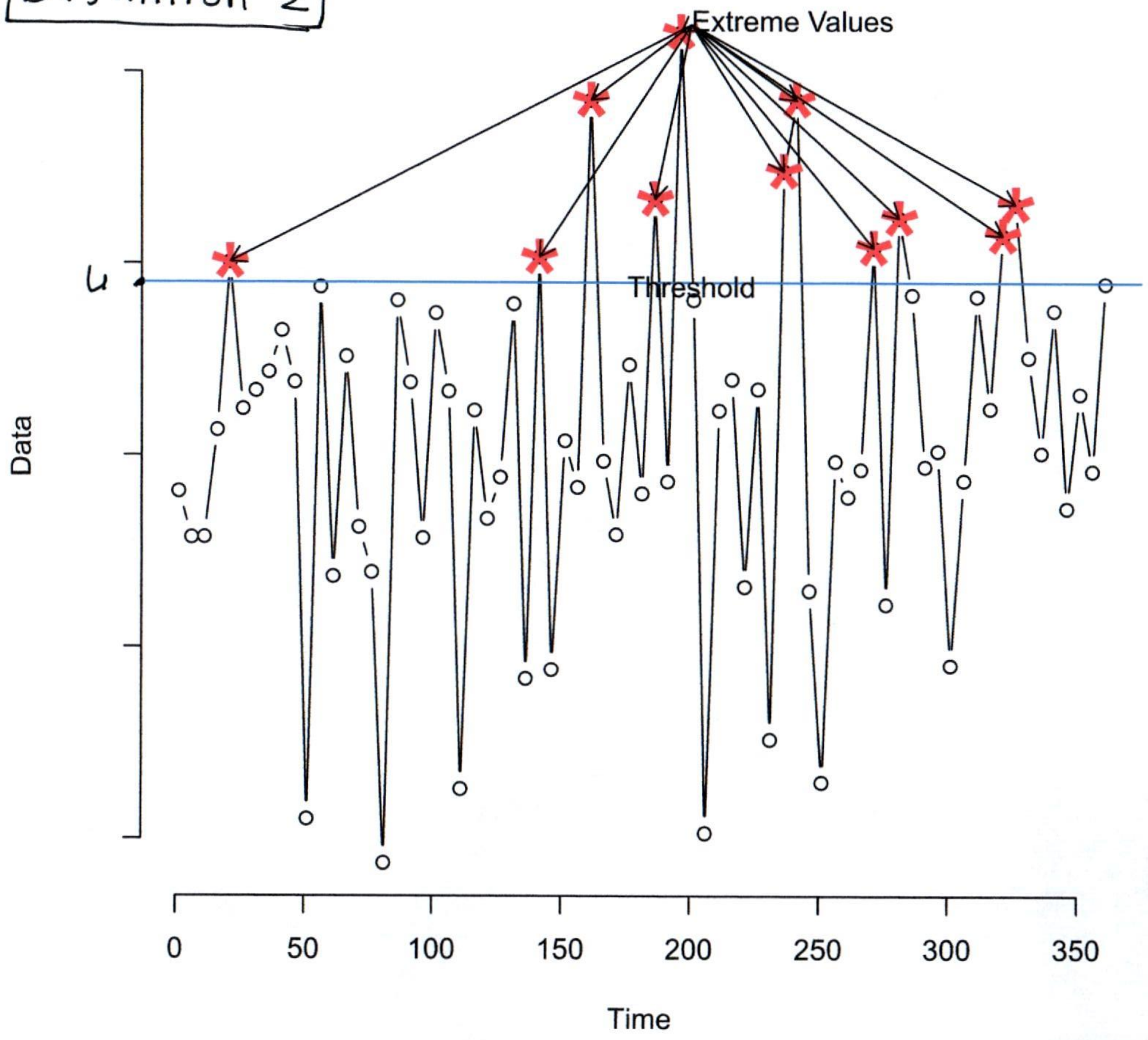
$$\left.\begin{matrix}\hat{\mu}\\ \hat{\sigma}\end{matrix}\right\} = \text{parametric estimators}$$

Hence,

$$\widehat{VaR}_p(X) = \hat{\mu} + \frac{\hat{\sigma}}{\hat{\xi}}\left[(-\log p)^{-\hat{\xi}} - 1\right]$$

is a semi-parametric estimate of VaR.

Definition 2



2) <u>GP</u> <u>method</u>

Suppose the data follow definition 2 of extreme values. Then the CDF

$$F(x) = 1 - q\left(1 + \xi\frac{x-u}{\sigma}\right)^{-\frac{1}{\xi}}$$

for $x > u$, where $q = P(X > u)$. Set

$$F(x) = p$$

$$\Leftrightarrow \quad 1 - q\left(1+\xi\frac{x-u}{\sigma}\right)^{-\frac{1}{\xi}} = p$$

$$\Leftrightarrow \quad x = u + \frac{\sigma}{\xi}\left[\left(\frac{1-p}{q}\right)^{-\xi} - 1\right]$$

$$\Leftrightarrow \quad VaR_p(X) = u + \frac{\sigma}{\xi}\left[\left(\frac{1-p}{q}\right)^{-\xi} - 1\right].$$

(i) estimator ξ as

$$\boxed{\hat{\xi} = \frac{1}{k}\sum_{i=1}^{k} \log\frac{x_{(i)}}{x_{(k+1)}} \qquad 1 \leq k \leq n}$$

Hill's estimator

where $x_{(1)} \geq x_{(2)} \geq \cdots \geq x_{(n)}$ are data arranged in decreasing order

(ii) estimate $\hat{\sigma}$ as the MLE.

Hence,

$$\widehat{VaR}_p(X) = u + \frac{\hat{\sigma}}{\hat{\xi}}\left[\left(\frac{1-p}{q}\right)^{-\hat{\xi}} - 1\right]$$

is a semi-parametric estimate of VaR.