# How to choose $a_n$ and $b_n$

I : $a_n = \gamma\left(F^{-1}\left(1-\frac{1}{n}\right)\right)$,

$b_n = F^{-1}\left(1-\frac{1}{n}\right)$

II : $a_n = F^{-1}\left(1-\frac{1}{n}\right)$,

$b_n = 0$

III : $a_n = w(F) - F^{-1}\left(1-\frac{1}{n}\right)$,

$b_n = w(F)$

$F^{-1}(\cdot)$ denotes the inverse function of $F(\cdot)$.

Example I $\quad F(x) = 1 - e^{-x}, \; x > 0$

condition I holds with $\gamma(t) \equiv 1$.

$$\Rightarrow \quad a_n = \gamma\left(F^{-1}\left(1 - \tfrac{1}{n}\right)\right)$$
$$= 1$$
$$\Rightarrow \quad F(x) = 1 - \frac{1}{n}$$
$$\Rightarrow \quad 1 - e^{-x} = 1 - \frac{1}{n}$$
$$\Rightarrow \quad e^{-x} = \frac{1}{n}$$
$$\Rightarrow \quad -x = -\log n$$
$$\Rightarrow \quad x = \log n$$
$$\Rightarrow \quad b_n = \log n$$

By the ETT,

$$\left[F(x + \log n)\right]^n \longrightarrow e^{-e^{-x}}$$

as $n \longrightarrow \infty$.

Example 2 $\quad F(x) = 1 - \frac{1}{x}$

condition (II) holds.

$$F(x) = 1 - \frac{1}{n}$$

$$\Rightarrow \quad 1 - \frac{1}{x} = 1 - \frac{1}{n}$$

$$\Rightarrow \quad x = n$$

$$\Rightarrow \quad a_n = n$$

$$b_n = 0$$

By the ETT,

$$[F(nx)]^n \longrightarrow \begin{cases} 0 & x < 0 \\ e^{-x^{-1}} & x \geq 0 \end{cases}$$

as $n \longrightarrow \infty$

## Example 3

$$F(x) = x, \quad 0 < x < 1$$

condition III holds with $\alpha = 1$

$$F(x) = 1 - \frac{1}{n}$$

$$\Rightarrow \quad x = 1 - \frac{1}{n}$$

$$\Rightarrow \quad F^{-1}\left(1 - \frac{1}{n}\right) = 1 - \frac{1}{n}.$$

$$\Rightarrow \quad a_n = 1 - \left(1 - \frac{1}{n}\right) = \frac{1}{n}$$

$$b_n = 1$$

By the ETT,

$$\left[F\left(\frac{x}{n} + 1\right)\right]^n \longrightarrow \begin{cases} e^x & x < 0 \\ 1 & x \geq 0 \end{cases}$$

as $n \longrightarrow \infty$.