

MATH11712 Statistics I
Semester 2, 2022 / 2023
Example Sheet 12

1. A consumer group is testing the heating capacity of two types of camp stove. For each stove the investigators record the time taken (in minutes) to bring one litre of water from 10 degrees Centigrade to boiling (at sea level). Two competing stove types are under consideration. Ten stoves of Type A and twelve stoves of Type B are selected at random and tested. The following results are obtained:

$$\text{Type A: } n_1 = 10, \quad \bar{x}_1 = 11.4$$

$$\text{Type B: } n_2 = 12, \quad \bar{x}_2 = 9.9.$$

Assume that the boiling times for each stove are normally distributed, with known variance $\sigma_1^2 = 6.25$ and $\sigma_2^2 = 9.0$ for Type A and Type B respectively.

- (i) Test the following hypotheses at the 5% significance level:

$$H_0 : \mu_1 - \mu_2 = 0 \text{ vs } H_1 : \mu_1 - \mu_2 > 0,$$

and report your conclusions.

- (ii) What is the probability that this test will correctly reject H_0 if the true value of $\mu_1 - \mu_2$ is 2.0?

2. To compare the wearing qualities of two types of car tyre, Type A and Type B, twenty five randomly selected tyres of Type A and twenty five randomly selected tyres of Type B were each subjected to a laboratory wear test. The amount of wear (in mm) at the end of the test was recorded for each tyre with the following results:

$$\text{Tyre A: } n_1 = 25, \quad \bar{x}_1 = 10.24$$

$$\text{Tyre B: } n_2 = 25, \quad \bar{x}_2 = 9.76.$$

Assume that for both types the wear has a normal distribution with common variance $\sigma^2 = 1.742$.

- (i) Test at the 5% significance level whether there is a difference in the mean tyre wear for the types A and B.
- (ii) What is the probability that the above test will incorrectly fail to reject the null hypothesis if the true difference in means is equal to 1.0?
3. An experiment was conducted to compare the mean length of time required for the bodily absorption of Drug A and Drug B. Two random samples of patients were obtained. The first sample contained thirty three people, who were assigned to receive Drug A. The second sample contained thirty seven people, who were

assigned to Drug B. For each patient, the response recorded was the length of time (in minutes) for the drug to reach a specified concentration in the blood.

The data has been summarized as follows:

$$\text{Drug A: } n_1 = 33, \quad \bar{x}_1 = 28.2, \quad s_1^2 = 16.36$$

$$\text{Drug B: } n_2 = 37, \quad \bar{x}_2 = 30.7, \quad s_2^2 = 18.92.$$

Assuming that the data are normally distributed, test whether the data provide sufficient evidence at the 5% significance level to indicate a difference in mean times to absorption for the two drugs.

You should carry out the test under the assumption that (i) $\sigma_1^2 \neq \sigma_2^2$ and (ii) $\sigma_1^2 = \sigma_2^2$, where σ_1^2 and σ_2^2 denote the variance of the absorption time for Drug A and B respectively.

4. A random sample of $n_1 = 49$ measurements from a particular population was found to have a sample mean of $\bar{x}_1 = 9.8$ and sample standard deviation of $s_1 = 2.9$. A second random sample of $n_2 = 64$ measurements was found to have a sample mean of $\bar{x}_2 = 11.9$ and a sample standard deviation of $s_2 = 4.2$. The second sample was independent of the first. Using an approximate significance level of 1%, test the claim that the population means are different.
5. A study was carried out in the US to look at the extent of the belief in extraterrestrials amongst the adult population. A random sample of $n_1 = 100$ people was obtained from the population of adult Americans who did not attend college, and a second random sample of $n_2 = 100$ was obtained from the population of adult Americans who did attend college.

It was found that, of those sampled, the number believing in extraterrestrials was $r_1 = 37$ among those who did not attend college, and $r_2 = 47$ among those who did attend college.

Do these results indicate that the proportion of people in the population who believe in extraterrestrials is higher among those who have attended college than among those who have not? You should conduct an appropriate hypothesis test at an approximate significance level of 5%.