MATH11712 Statistics I
Semester 2, 2022 / 2023
Example Sheet 9

1. A random sample of $n = 81$ students took an achievement test and had their scores recorded. The sample mean of the scores was found to be 74.6 and the sample standard deviation was 11.3. Find a 90% confidence interval for the mean score of all students.

2. A random sample of size $n = 30$ is taken from a $N(\mu, \sigma^2)$ distribution. The observed values of $x_1, \ldots, x_{30}$ give the results $\sum_{i=1}^{30} x_i = 1568.45$ and $\sum_{i=1}^{30} x_i^2 = 83006.73$. Find 95% confidence intervals for

(i) μ, if it is known that $\sigma^2 = 30$,

(ii) μ and σ^2, if both μ and σ^2 are unknown.

3. In a random sample of $n = 500$ households, 80 were found to have three or more TV sets. Find an approximate 90% CI for the proportion of all households having three or more TV sets.

4. Let $X_1, \ldots, X_n$ be a random sample from a $N(\mu, \sigma^2)$ distribution where μ is unknown but σ^2 is known. What is the width of the $100(1-\alpha)\%$ confidence interval for μ? Which variable quantities affect this width?

If $\sigma = 10$, tabulate the widths of confidence intervals for μ for $n = 5, 15, 30, 50, 100$ when (i) $\alpha = 0.1$ and (ii) $\alpha = 0.01$.

Illustrate your results on a sketch scatterplot with sample size on the horizontal axis, and CI width on the vertical axis. You should notice that the width decreases at the rate $1/\sqrt{n}$.

5. A psychologist knows that measurements of reaction time are normally distributed with $\sigma = 0.05$. What are the numbers of measurements she must take so that a (i) 95% and (ii) 99% CI for the mean reaction time has a width not exceeding 0.02?

6. It is believed that, in a certain adult male population, systolic blood pressure is normally distributed but both the mean μ and variance σ^2 are unknown. Suppose that a random sample of $n = 30$ from this population gives the results that $\bar{x} = 127.442$ and $s^2 = 228.661$. Find a 98% CI for μ and a 98% CI for σ^2.

7. Suppose that $X_1, \ldots, X_n \sim N(\mu_X, \sigma_X^2)$ independently, and let $Y_i = aX_i + b$. Note that $Y_1, \ldots, Y_n \sim N(\mu_Y, \sigma_Y^2)$.

(i) Show that $\mu_Y = a\mu_X + b$ and $\sigma_Y^2 = a^2\sigma_X^2$.

(ii) Show that $\bar{Y} = a\bar{X} + b$ and $S_Y^2 = a^2 S_X^2$, where S_X^2 and S_Y^2 denote the sample variance of $X_1, \ldots, X_n$ and $Y_1, \ldots, Y_n$ respectively.

Let $[L_X, U_X]$ denote the $100(1-\alpha)\%$ confidence interval for μ_X and $[L_Y, U_Y]$ denote the $100(1-\alpha)\%$ confidence interval for μ_Y.

(iii) Show that $[L_Y, U_Y] = [aL_X + b, aU_X + b]$.

Let $[V_X, W_X]$ denote the $100(1-\alpha)\%$ confidence interval for σ_X^2 and $[V_Y, W_Y]$ denote the $100(1-\alpha)\%$ confidence interval for σ_Y^2.

(iv) Show that $[V_Y, W_Y] = [a^2 V_X, a^2 W_X]$.