Extremal Types Theorem (ETT)

Suppose $X_1, \ldots, X_n$ are independent and identical with CDF $F(\cdot)$. If there exists $a_n > 0$ and $b_n \in (-\infty, \infty)$ such that

$$(*) = G(x)$$

for some non-degenerate CDF $G(x)$ then it must be of the same type as

$$\text{I} : \Lambda(x) = e^{-e^{-x}}, \quad -\infty < x < \infty$$

Gumbel type

$$\text{II} : \Phi_\alpha(x) = \begin{cases} 0, & \text{if } x < 0 \\ e^{-x^{-\alpha}}, & \text{if } x \geq 0 \end{cases}$$

Fréchet type

$$\text{III} : \Psi_\alpha(x) = \begin{cases} e^{-(-x)^\alpha}, & \text{if } x < 0 \\ 1, & \text{if } x \geq 0 \end{cases}$$

Weibull type

Definition of "same type"

Two CDFs G_1 and G_2 are of the same type if

$$G_2(x) = G_1(ax+b)$$

for all x, where $a > 0$ and $b \in \mathbb{R}$.

Example 1

$$G_1(x) = e^{-e^{-x}}$$

$$G_2(x) = e^{-e^{-2x-4}}$$

$\Rightarrow$ $a = 2$, $b = 4$

$\Rightarrow$ G_1 & G_2 are of the same type

Example 2

$$G_1(x) = e^{-\frac{1}{x}}$$

$$G_2(x) = e^{-\frac{1}{3-2x}}$$

$\Rightarrow$ $a = -2$, $b = 3$

$\Rightarrow$ G_1 & G_2 are not of the same type