A differentiation trick

$$\frac{d x^a}{da} = x^a \log x \qquad (1)$$

$$\Rightarrow \frac{dx^a}{da}\Big|_{a=0} = x^0 \log x = \log x$$

$$\Rightarrow \frac{\int x^{\alpha}}{\int a} \Big|_{\alpha=0} = \log x$$

$$(1) \Rightarrow \frac{\int^2 x^a}{\int a^2} = \frac{\int x^a}{\int a} \log x = x^a (\log x)^2$$

$$\Rightarrow \frac{d^2x^a}{da^2}\Big|_{a=0} = x^0 (\log x)^2 = (\log x)^2$$

$$\Rightarrow \frac{\int_{a}^{2} x^{a}}{\int_{a}^{2} a^{2}} \Big|_{a=0} = (\log x)^{2}$$

In general,

$$\frac{\int_{\alpha}^{\alpha} \int_{\alpha}^{\alpha} d\alpha}{\int_{\alpha}^{\alpha} d\alpha} = (\log x)^{\alpha}$$