## Definition of bivariate extreme value

Suppose $(X, Y)$ is a random vector. Suppose too

- $X_1, \dots, X_n$ are data on $X$
- $Y_1, \dots, Y_n$ are data on $Y$

The most common definition of a bivariate extreme value is

$$(M_{n,1}, M_{n,2}) = [\max(X_1, \dots, X_n), \max(Y_1, \dots, Y_n)]$$

But this may not be an actual observation.

Ex 1 Suppose the data are

| | | | | | |
|---|---|---|---|---|---|
| X | -5 | 0 | 1 | 8 | 2 |
| Y | 3 | 2 | 9 | 1 | 3 |

The bivariate extreme value is

$$(M_{n,1}, M_{n,2}) = (8, 9)$$

This is not an actual observation.

## Joint CDF $\left(\dfrac{M_{n,1}-b_n}{a_n}, \dfrac{M_{n,2}-d_n}{c_n}\right)$

Suppose $(X_1, Y_1), (X_2, Y_2), \ldots, (X_n, Y_n)$ are IID with joint CDF $F$. The required CDF is

$$P\left(\frac{M_{n,1}-b_n}{a_n} \leq x, \frac{M_{n,2}-d_n}{c_n} \leq y\right)$$

$$= P\left(M_{n,1} \leq a_n x + b_n, M_{n,2} \leq c_n y + d_n\right)$$

$$= P\Big(\max(X_1, \ldots, X_n) \leq a_n x + b_n, \max(Y_1, \ldots, Y_n) \leq c_n y + d_n\Big)$$

$$= P\Big(X_1 \leq a_n x + b_n, \ldots, X_n \leq a_n x + b_n, Y_1 \leq c_n y + d_n, \ldots, Y_n \leq c_n y + d_n\Big)$$

$$= P\Big(X_1 \leq a_n x + b_n, Y_1 \leq c_n y + d_n, \; \vdots \; X_n \leq a_n x + b_n, Y_n \leq c_n y + d_n\Big)$$

$\boxed{\text{Independence}}$

$$= P(X_1 \leq a_n x + b_n, Y_1 \leq c_n y + d_n) \; \vdots \; P(X_n \leq a_n x + b_n, Y_n \leq c_n y + d_n)$$

$$= F(a_n x + b_n, c_n y + d_n)$$

$$\vdots$$

$$F(a_n x + b_n, c_n y + d_n)$$

$$= \left[F(a_n x + b_n, c_n y + d_n)\right]^n \quad \cdots (*)$$

What is the limit of $(*)$ as $n \to \infty$?

$$\boxed{\text{If } (*) \longrightarrow G(x, y) \text{ as } n \to \infty \text{ for a non-degenerate CDF } G \text{ then possible forms for } G \text{ can be uncountably infinite.}}$$