Case 6 of Portfolio Theory

[$X_1, \ldots, X_N$ are dependent RVs & N is a RV independent $X_1, X_2, \ldots$]

a) The CDF of S' is

$$F_{S'}(s) = P(S' \leq s)$$

$$= \sum_{n=1}^{\infty} P(S' \leq s \mid N=n)\, P(N=n)$$

Total Prob Rule

$$= \sum_{n=1}^{\infty} \left[\int \cdots \int_{\sum_{i=1}^{n} x_i \leq s} f_{X_1, \ldots, X_n}(x_1, \ldots, x_n)\, dx_n \ldots dx_1 \right] \cdot P(N=n)$$

Case 5

The PDF of S' is

$$f_{S'}(s) = \frac{d}{ds} F_{S'}(s)$$

b) The CDF of U is

$$F_U(u) = P(U \leq u)$$

$$= \sum_{n=1}^{\infty} P(U \leq u \mid N=n)\, P(N=n)$$

$\boxed{\text{Total Prob Rule}}$

$$= \sum_{n=1}^{\infty} \left[1 - \bar{F}_{X_1,\ldots,X_n}(u,\ldots,u)\right] P(N=n)$$

$$= \sum_{n=1}^{\infty} P(N=n) - \sum_{n=1}^{\infty} \bar{F}_{X_1,\ldots,X_n}(u,\ldots,u)\, P(N=n)$$

$$= 1 - \sum_{n=1}^{\infty} \bar{F}_{X_1,\ldots,X_n}(u,\ldots,u)\, P(N=n).$$

The PDF of U is

$$f_U(u) = \frac{d}{du} F_U(u)$$

$$= -\sum_{n=1}^{\infty} \left[\frac{d}{du} \bar{F}_{X_1,\ldots,X_n}(u,\ldots,u)\right] P(N=n).$$

c) The CDF of V is

$$F_V(v) = P(V \leq v)$$

$$= \sum_{n=1}^{\infty} P(V \leq v \mid N=n)\, P(N=n)$$

$\boxed{\text{Total Prob Rule}}$

$$= \sum_{n=1}^{\infty} F_{X_1,\dots,X_n}(v,\dots,v)\, P(N=n).$$

$\boxed{\text{Case 5}}$

The PDF of V is

$$f_V(v) = \frac{d}{dv} F_V(v)$$

$$= \sum_{n=1}^{\infty} \left[\frac{d}{dv} F_{X_1,\dots,X_n}(v,\dots,v)\right] P(N=n).$$