Case 4 of Portfolio Theory

[$X_1, \ldots, X_N$ are INID & N is a RV independent $X_1, X_2, \ldots$]

a) The CDF of S' is

$$F_{S'}(s) = P(S' \leq s)$$

$$= \sum_{n=1}^{\infty} P(S' \leq s \mid N=n)\, P(N=n)$$

Total Prob Rule

$$= \sum_{n=1}^{\infty} \left[\int_{\sum_{i=1}^{n} x_i \leq s} \prod_{i=1}^{n} f_{X_i}(x_i)\, dx_n \cdots dx_1 \right] P(N=n)$$

Case 3

The PDF of S' is

$$f_{S'}(s) = \frac{d}{ds} F_{S'}(s).$$

b) The CDF of U is

$$F_U(u) = P(U \leq u)$$

$$= \sum_{n=1}^{\infty} P(U \leq u \mid N=n)\, P(N=n)$$

Total Prob Rule

$$= \sum_{n=1}^{\infty} \left\{ 1 - \prod_{i=1}^{n} [1 - F_{X_i}(u)] \right\} P(N=n)$$

$$= \sum_{n=1}^{\infty} P(N=n) - \sum_{n=1}^{\infty} \prod_{i=1}^{n} [1 - F_{X_i}(u)]\, P(N=n)$$

$$= 1 - \sum_{n=1}^{\infty} \prod_{i=1}^{n} [1 - F_{X_i}(u)]\, P(N=n).$$

The PDF of U is

$$f_U(u) = \sum_{n=1}^{\infty} \sum_{i=1}^{n} f_{X_i}(u) \prod_{\substack{j=1 \\ j \neq i}}^{n} [1 - F_{X_j}(u)]\, P(N=n)$$

Product Rule

c) The CDF of V is

$$F_V(v) = P(V \leq v)$$

$$= \sum_{n=1}^{\infty} P(V \leq v \mid N=n)\, P(N=n)$$

Total Prob Rule

$$= \sum_{n=1}^{\infty} \left[\prod_{i=1}^{n} F_{X_i}(v) \right] P(N=n).$$

Case 3

The PDF of V is

$$f_V(v) = \frac{d}{dv} F_V(v)$$

$$= \sum_{n=1}^{\infty} \left[\sum_{i=1}^{\infty} f_{X_i}(v) \prod_{\substack{j=1 \\ j \neq i}}^{n} F_{X_j}(v) \right] P(N=n)$$

Product Rule