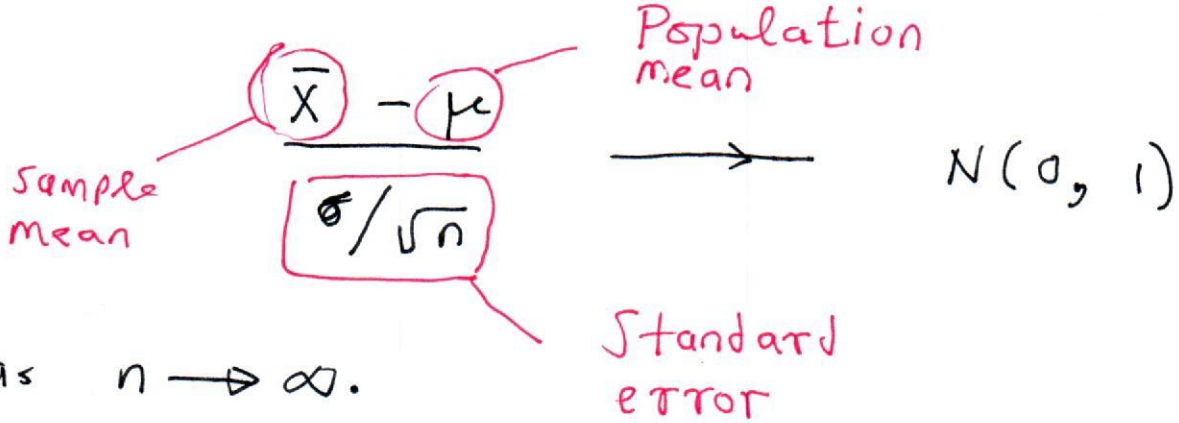
# Central Limit Theorem

Suppose $X_1, \ldots, X_n$ is a random sample from a population with $E(X) = \mu$ and $\text{Var}(X) = \sigma^2$. Then




as $n \to \infty$.

# A model based on CLT

Let $n$ = sample size. $n$ can never be equal to $\infty$.

Suppose $n$ is large enough such that CLT holds. In other words,

$$\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \boxed{\approx} N(0, 1)$$

$\approx$: approximation

$$\Rightarrow P\left(\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \leq x\right) \approx \Phi(x)$$

$$\Rightarrow P\left(\bar{X} \leq \boxed{\mu + \frac{\sigma}{\sqrt{n}} x}\right) \approx \Phi(x)$$

$\boxed{\mu + \frac{\sigma}{\sqrt{n}} x} = y$

$$\Rightarrow P(\bar{X} \leq y) \approx \Phi\left(\frac{\sqrt{n}}{\sigma}(y - \mu)\right)$$

$$\Rightarrow P\left(\frac{1}{n}\sum_{i=1}^{n} X_i \leq y\right) \approx \Phi\left(\frac{\sqrt{n}}{\sigma}(y - \mu)\right)$$

$$\Rightarrow P\left(\sum_{i=1}^{n} X_i \leq \boxed{ny}\right) \approx \Phi\left(\frac{\sqrt{n}}{\sigma}(y - \mu)\right)$$

$\boxed{ny} = z$

$$\Rightarrow \boxed{P\left(\sum_{i=1}^{n} X_i \leq z\right) \approx \Phi\left(\frac{\sqrt{n}}{\sigma}\left(\frac{z}{n} - \mu\right)\right)}$$

## Example 3 from Binomial distribution (Contd)

Suppose $X_1, \ldots, X_n$ are independent and identical Bernoulli(0.2) RVs.

$$\Rightarrow \sum_{i=1}^{1000} X_i \sim \text{Bin}(1000, 0.2)$$

$$\Rightarrow P(\text{Bin}(1000, 0.2) \leq z) \approx \Phi\left(\frac{\sqrt{1000}}{\sigma}\left(\frac{z}{1000} - \mu\right)\right) \quad \ldots (*)$$

We wanted to find

$$P(100 \leq \text{Bin}(1000, 0.2) \leq 200)$$

$$= P(\text{Bin}(1000, 0.2) \leq 200) - P(\text{Bin}(1000, 0.2) \leq 99)$$

$$\overset{(*)}{\approx} \Phi\left(\frac{\sqrt{1000}}{\sigma}\left(\frac{200}{1000} - \mu\right)\right) - \Phi\left(\frac{\sqrt{1000}}{\sigma}\left(\frac{99}{1000} - \mu\right)\right)$$

$$\mu = E(X_i) = 0.2$$

$$\sigma^2 = Var(X_i) = 0.2 \times 0.8$$

$$\sigma = \sqrt{0.2 \times 0.8}$$

$$= \Phi\left(\frac{\sqrt{1000}}{\sqrt{0.2 \times 0.8}}\left(0.2 - 0.2\right)\right)$$

$$- \Phi\left(\frac{\sqrt{1000}}{\sqrt{0.2 \times 0.8}}\left(0.099 - 0.2\right)\right)$$

$$= 0.5 - \Phi\left(\frac{\sqrt{1000}}{\sqrt{0.2 \times 0.8}}\left(0.099 - 0.2\right)\right)$$

= calculate using normal table.