Confidence Intervals

What is the temperature at Manchester?

$$12^\circ c$$

Better to say

$$[11.9^\circ c, 12.2^\circ c]$$

That is why we need confidence intervals.

Definitions

Suppose $X_1, \ldots, X_n$ are IID data from a distribution parameterised by θ.

1) A <u>$100(1-\alpha)\%$</u> <u>confidence</u> <u>interval</u> for θ is

$$[L(X_1, \ldots, X_n), \; U(X_1, \ldots, X_n)]$$

such that

$$P[L(X_1, \ldots, X_n) \leq \theta \leq U(X_1, \ldots, X_n)] = 1 - \alpha$$

2) <u>Confidence</u> <u>limits</u> are

$$L(X_1, \ldots, X_n)$$

and

$$U(X_1, \ldots, X_n).$$

3) <u>Coverage</u> <u>probability</u> is

$$P[L(X_1, \ldots, X_n) \leq \theta \leq U(X_1, \ldots, X_n)]$$

4) <u>Coverage</u> <u>length</u> is

$$U(X_1, \ldots, X_n) - L(X_1, \ldots, X_n)$$