An example where ETT fails

(an example where none of conditions I - III will be satisfied).

Suppose $F(x) = 1 - \dfrac{1}{\log x} \quad , \quad x > e$

$$F(x) = 1$$

$$\Rightarrow \quad 1 - \frac{1}{\log x} = 1$$

$$\Rightarrow \quad \frac{1}{\log x} = 0$$

$$\Rightarrow \quad \log x = +\infty$$

$$\Rightarrow \quad x = +\infty$$

$$\Rightarrow \quad \omega(F) = +\infty$$

I : $\displaystyle \lim_{t \to \infty} \frac{1 - F(t + x\gamma(t))}{1 - F(t)}$

$$= \lim_{t \to \infty} \frac{\cancel{1} - \left[\cancel{1} - \dfrac{1}{\log(t + x\gamma(t))}\right]}{\cancel{1} - \left[\cancel{1} - \dfrac{1}{\log t}\right]}$$

$$= \lim_{t \to \infty} \frac{\log t}{\log(t + x\gamma(t))}$$

$$\neq e^{-x}$$

$\Rightarrow$ Condition I fails

II : $w(F) = +\infty$ ✓

$$\lim_{t \to \infty} \frac{1 - F(tx)}{1 - F(t)}$$

$$= \lim_{t \to \infty} \frac{\cancel{1} - \left[\cancel{1} - \frac{1}{\log(tx)}\right]}{\cancel{1} - \left[\cancel{1} - \frac{1}{\log t}\right]}$$

$$= \lim_{t \to \infty} \frac{\log t}{\log(tx)}$$

$$= \lim_{t \to \infty} \frac{\log t}{\log t + \log x}$$

$$= \lim_{t \to \infty} \frac{1}{1 + \frac{\log x}{\log t}} \quad \frac{\log x}{\log t} \to 0$$

$$= 1 \neq x^{-\alpha}$$

$\Rightarrow$ Condition II fails

$\underline{\text{III}}$ $w(F) < \infty$, which is not the case

condition $\underline{\text{III}}$ fails

All three conditions I-$\underline{\text{III}}$ fail.
Hence, the ETT does not hold.